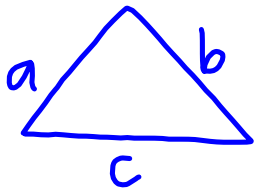


## **Theorem 5-13** **Triangle Inequality Theorem**

The sum of the lengths  
of any two sides of a triangle  
is greater than the length  
of the third side.



$$a + b > c$$

$$b + c > a$$

$$c + a > b$$

### **Example 1**

Determine if the given values would make a triangle.

a) 4, 6, 9

$$4 + 6 > 9 \checkmark$$

$$6 + 9 > 4 \checkmark$$

$$9 + 4 > 6 \checkmark$$

YES

b) 5, 10, 16

$$5 + 10 \not> 16$$

NO

c) 21, 34, 55

$$21 + 34 \not> 55$$

NO

Example 2

If Mrs. Bailey gave Susan four pieces of copper tubing measuring 6 m, 7 m, 9 m, and 16 m, how many different triangles could Susan make?

$$\begin{array}{l} 6, 7, 9 \\ 6+7 > 9 \checkmark \\ 7+9 > 6 \checkmark \\ 9+6 > 7 \checkmark \\ \triangle \end{array}$$

$$\begin{array}{l} 7, 9, 16 \\ 7+9 < 16 \\ \text{not } \triangle \end{array}$$

$$\begin{array}{l} 6, 9, 16 \\ 6+9 < 16 \\ \text{not } \triangle \end{array}$$

$$\begin{array}{l} 6, 7, 16 \\ 6+7 < 16 \\ \text{not } \triangle \end{array}$$

1 triangle

Example 3

If 18, 45, 21, and 52 represent lengths of segments, what is the probability that a triangle can be formed if three of these numbers are chosen at random as lengths of the sides?

$$\begin{array}{l} 18, 45, 21 \\ 18+45 > 21 \checkmark \\ 45+21 > 18 \checkmark \\ 21+18 < 45 \\ \text{not } \triangle \end{array}$$

$$\begin{array}{l} 45, 21, 52 \\ 45+21 > 52 \checkmark \\ 21+52 > 45 \checkmark \\ 52+45 > 21 \checkmark \\ \triangle \end{array}$$

$$\begin{array}{l} 18, 21, 52 \\ 18+21 < 52 \\ \text{not } \triangle \end{array}$$

$$\begin{array}{l} 18, 45, 52 \\ 18+45 > 52 \checkmark \\ 45+52 > 18 \checkmark \\ 52+18 > 45 \checkmark \\ \triangle \end{array}$$

$$\frac{2}{4} = \frac{1}{2} \text{ or } 50\%$$

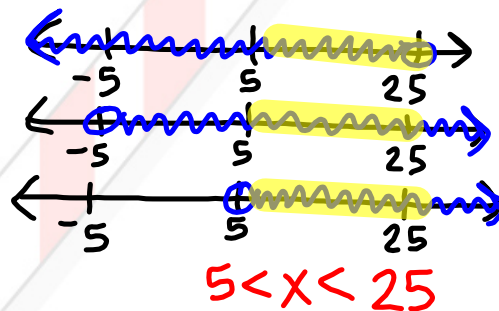
Example 4      10, 15, ?

If the lengths of two sides of a triangle are 10 centimeters and 15 centimeters, between what two numbers must the measure of the third side fall?

$$\begin{aligned} 10 + 15 &> x \\ 25 &> x \\ x &< 25 \end{aligned}$$

$$\begin{array}{r} 15 + x > 10 \\ -15 \quad -15 \\ \hline x > -5 \end{array}$$

$$\begin{array}{r} x + 10 > 15 \\ -10 \quad -10 \\ \hline x > 5 \end{array}$$

Example 5

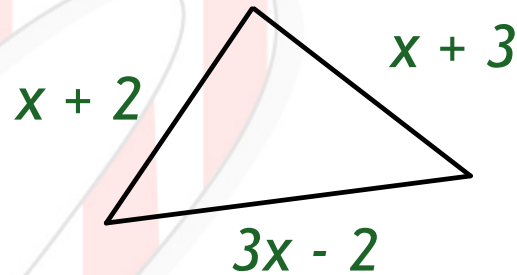
The lengths of two sides of a triangle are 8 and 13. What are the possible lengths of the third side?

$$8, 13, ?$$

$$5 < x < 21$$

**Example 6**

Use the Triangle Inequality to find the possible values of  $x$ .

**Example 7**

Use the Triangle Inequality to find the possible values of  $x$ .

A triangle with side lengths labeled  $3x - 1$ ,  $x + 4$ , and  $x + 2$ .

$$(3x - 1) + (x + 4) > x + 2$$

$$\frac{4x + 3}{-x} > \frac{x + 2}{-x}$$

$$\frac{3x - 3}{3} > \frac{2}{3}$$

$$\frac{3x}{3} > \frac{2 + 3}{3}$$

$$x > \frac{5}{3}$$

$$(3x - 1) + (x + 2) > x + 4$$

$$\frac{4x + 1}{-x} > \frac{x + 4}{-x}$$

$$\frac{3x + 1}{-1} > \frac{4}{-1}$$

$$\frac{3x}{3} > \frac{3}{3}$$

$$x > 1$$

$$(x + 4) + (x + 2) > 3x - 1$$

$$\frac{2x + 6}{-3x} > \frac{3x - 1}{-3x}$$

$$\frac{-x + 6}{-6} > \frac{-1}{-6}$$

$$\frac{-x}{-1} > \frac{-7}{-1}$$

$$x < 7$$

Number line diagram showing the intersection of  $x > 1$  and  $x < 7$ . The number line has points  $-\frac{1}{3}$ ,  $1$ , and  $7$ . Blue arrows point right from  $1$  and left from  $7$ . The intersection is shaded yellow and labeled  $1 < x < 7$  in a red box.