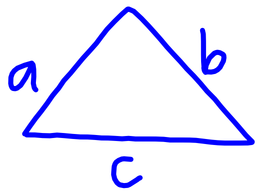
Theorem 5-13

Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.



$a + b > c$

$b + c > a$

$c + a > b$

Example 1

Determine if the given values would make a triangle.

a) 4, 6, 9

$4+6>9$ ✓

$6+9>4$ ✓

$9+4>6$ ✓

YES

b) 5, 10, 16

$5+10 \not> 16$

NO

c) 21, 34, 55

$21+34 \not> 55$

NO

Example 2

If Mrs. Bailey gave Susan four pieces of copper tubing measuring 6 m, 7 m, 9 m, and 16 m, how many different triangles could Susan make?

- 6, 7, 9
  - $6+7>9$ ✓
  - $7+9>6$ ✓
  - $9+6>7$ ✓
  - $\triangle$
- 7, 9, 16
  - $7+9 \not> 16$
  - not $\triangle$
- 6, 9, 16
  - $6+9 \not> 16$
  - not $\triangle$
- 6, 7, 16
  - $6+7 \not> 16$
  - not $\triangle$

1 triangle

Example 3

If 18, 45, 21, and 52 represent lengths of segments, what is the probability that a triangle can be formed if three of these numbers are chosen at random as lengths of the sides?

- 18, 45, 21
  - $18+45>21$ ✓
  - $45+21>18$ ✓
  - $21+18 \not> 45$
  - not $\triangle$
- 45, 21, 52
  - $45+21>52$ ✓
  - $21+52>45$ ✓
  - $52+45>21$ ✓
  - $\triangle$
- 18, 21, 52
  - $18+21 \not> 52$
  - not $\triangle$
- 18, 45, 52
  - $18+45>52$ ✓
  - $45+52>18$ ✓
  - $52+18>45$ ✓
  - $\triangle$

$\frac{2}{4} = \frac{1}{2}$ or 50%

Example 4

10, 15, ?

If the lengths of two sides of a triangle are 10 centimeters and 15 centimeters, between what two numbers must the measure of the third side fall?

$10+15>x$

$25>x$

$x<25$

$15+x>10$

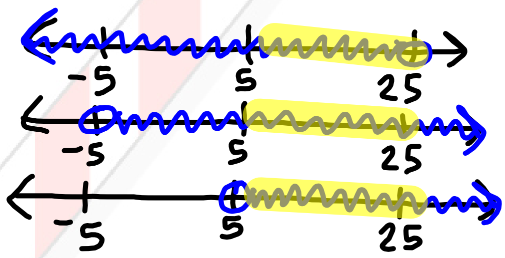
$-15 \quad -15$

$x>-5$

$x+10>15$

$-10 \quad -10$

$x>5$



$5<x<25$

Example 5

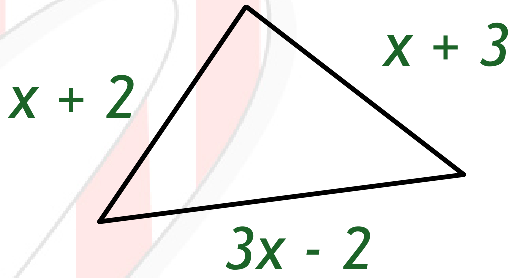
The lengths of two sides of a triangle are 8 and 13. What are the possible lengths of the third side?

8, 13, ?

$5<x<21$

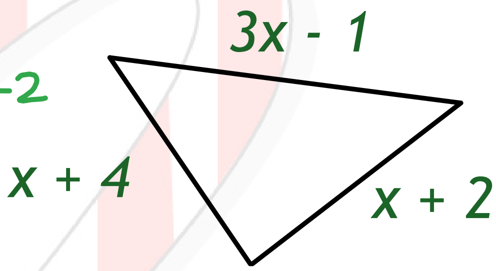
Example 6

Use the Triangle Inequality to find the possible values of x.



Example 7

Use the Triangle Inequality to find the possible values of x.



$(3x-1)+(x+4) > x+2$

$4x+3 > x+2$

$-x \quad -x$

$3x+3 > 2$

$-3 \quad -3$

$\frac{3x}{3} > \frac{-1}{3}$

$x > -\frac{1}{3}$

$(3x-1)+(x+2) > x+4$

$4x+1 > x+4$

$-x \quad -x$

$3x+1 > 4$

$-1 \quad -1$

$\frac{3x}{3} > \frac{3}{3}$

$x > 1$

$(x+4)+(x+2) > 3x-1$

$2x+6 > 3x-1$

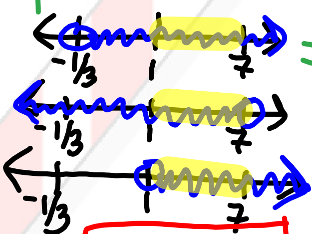
$-3x \quad -3x$

$-x+6 > -1$

$-6 \quad -6$

$\frac{-x}{-1} > \frac{-7}{-1}$

$x < 7$



$1 < x < 7$