7.3 Part 3 REMAINDER & FACTOR THEOREMS

REMAINDER THEOREM

If the polynomial P(x) is divided by x - c, then the remainder is the value P(c).

Example 1

Use synthetic division and the Remainder Theorem to evaluate P(c) if $P(x) = x^3 - 2x^2 - 5x + 10$ and $c = 1$.

$$\begin{array}{r|rrrr} 1 & 1 & -2 & -5 & 10 \\ & \downarrow & 1 & -1 & -6 \\ \hline & 1 & -1 & -6 & 4 \end{array}$$

remainder = 4

$P(1) = (1)^3 - 2(1)^2 - 5(1) + 10$

$P(1) = 1 - 2 - 5 + 10$

$P(1) = 4$ ← remainder

Example 2

Use synthetic division and the Remainder Theorem to evaluate P(c) if $P(x) = x^3 + 2x^2 - 7$ and $c = -2$.

$$\begin{array}{r|rrrr} -2 & 1 & 2 & 0 & -7 \\ & \downarrow & -2 & 0 & 0 \\ \hline & 1 & 0 & 0 & -7 \end{array}$$

remainder = -7

$P(-2) = (-2)^3 + 2(-2)^2 - 7$

$P(-2) = -8 + 8 - 7$

$P(-2) = -7$

remainder

Example 3

Find the remainder when $P(x) = 3x^3 + 4x^2 - 2x + 1$ is divided by $x - \frac{2}{3}$.

$$P\left(\tfrac{2}{3}\right) = 3\left(\tfrac{2}{3}\right)^3 + 4\left(\tfrac{2}{3}\right)^2 - 2\left(\tfrac{2}{3}\right) + 1$$
$$= 3\left(\tfrac{8}{27}\right) + 4\left(\tfrac{4}{9}\right) - \tfrac{4}{3} + 1$$
$$= \tfrac{8}{9} + \tfrac{16}{9} - \tfrac{12}{9} + \tfrac{9}{9}$$
$$P\left(\tfrac{2}{3}\right) = \tfrac{21}{9} = \tfrac{7}{3} \qquad \text{remainder} = \tfrac{7}{3}$$

FACTOR THEOREM

For a polynomial $P(x)$, $x - c$ is a factor if and only if $P(c) = 0$.

Example 4

Let $P(x) = x^3 - 7x + 6$. Show that $P(1) = 0$, and use this fact to factor $P(x)$ completely.

$$P(1) = (1)^3 - 7(1) + 6$$
$$P(1) = 1 - 7 + 6$$
$$P(1) = 0$$

↑ remainder

$$\begin{array}{c|cccc} 1 & 1 & 0 & -7 & 6 \\ & \downarrow & 1 & 1 & -6 \\ \hline & 1 & 1 & -6 & 0 \end{array}$$

$x = 1$

$$(x-1)(1x^2 + x - 6)$$

s1 p−6

$\frac{3}{1}$ $\frac{-2}{1}$

$$\boxed{P(x) = (x+1)(x+3)(x-2)}$$

Example 5

$x = -2$
$+2 \quad +2$
$x + 2 = 0$

use factor theorem or synthetic div.

Let $P(x) = 2x^3 - 3x^2 - 11x + 6$. Show that $P(-2) = 0$, and use this fact to write P(x) in factored form.

$$\begin{array}{r|rrrr} -2 & 2 & -3 & -11 & 6 \\ & \downarrow & -4 & 14 & -6 \\ \hline & 2 & -7 & 3 & 0 \end{array}$$

$(x+2)(2x^2 - 7x + 3)$

sum -7 product 6

$\frac{-3}{1} \quad \frac{-\cancel{6}}{\cancel{2}} \quad \frac{-1}{2}$

$$\boxed{(x+2)(x-3)(2x-1) = P(x)}$$

Example 6

Let $P(x) = x^4 - 6x^3 + 3x^2 + 26x - 24$. If $P(3) = 0$ and $P(-2) = 0$, write P(x) in factored form.

$$\begin{array}{r|rrrrr} 3 & 1 & -6 & 3 & 26 & -24 \\ & \downarrow & 3 & -9 & -18 & 24 \\ \hline & 1 & -3 & -6 & 8 & 0 \end{array}$$

$0 \leftarrow$ rem.

$(x-3)(x^3 - 3x^2 - 6x + 8)$

$$\begin{array}{r|rrrr} -2 & 1 & -3 & -6 & 8 \\ & \downarrow & -2 & 10 & -8 \\ \hline & 1 & -5 & 4 & 0 \end{array}$$

$0 \leftarrow$ rem.

$(x-3)(x+2)(x^2 - 5x + 4)$

sum -5 product 4

$\frac{-4}{1} \quad \frac{-1}{1}$

$$\boxed{(x-3)(x+2)(x-4)(x-1) = P(x)}$$

Example 7

Given the polynomial $2x^3 + x^2 - 13x + 6$ and the factor $x + 3$, find the remaining factors and write the polynomial in factored form.

$$\begin{array}{r|rrrr} -3 & 2 & 1 & -13 & 6 \\ & \downarrow & -6 & 15 & -6 \\ \hline & 2 & -5 & 2 & 0 \leftarrow \text{rem.} \end{array}$$

$(x+3)(2x^2-5x+2)$

sum -5 product 4

$\frac{-2}{1}$ $\cancel{\frac{-4}{2}}$ $\frac{-1}{2}$

$$\boxed{(x+3)(x-2)(2x-1)=P(x)}$$

Example 8

Given the polynomial $4x^3 + 13x^2 - 37x - 10$ and the factor $x - 2$, find the remaining factors and write the polynomial in factored form.

$$\begin{array}{r|rrrr} 2 & 4 & 13 & -37 & -10 \\ & \downarrow & 8 & 42 & 10 \\ \hline & 4 & 21 & 5 & 0 \leftarrow \text{rem.} \end{array}$$

$(x-2)(4x^2+21x+5)$

sum 21 product 20

$\frac{5}{1}$ $\cancel{\frac{20}{4}}$ $\frac{1}{4}$

$$\boxed{(x-2)(x+5)(4x+1)=P(x)}$$