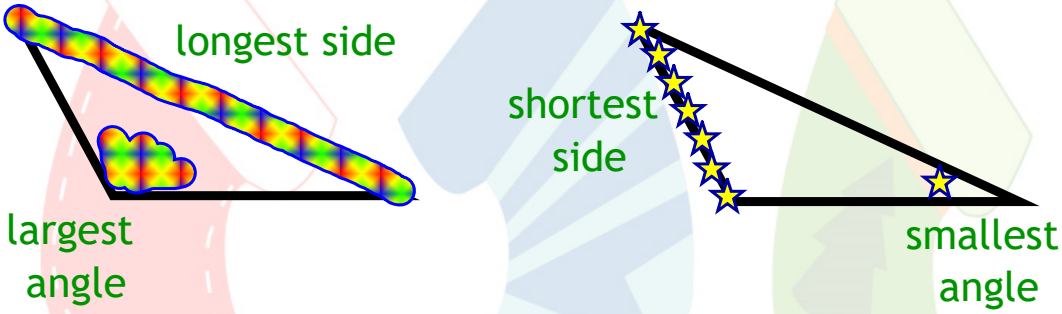
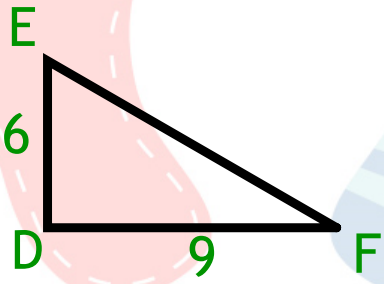
5.5 *USE INEQUALITIES IN A TRIANGLE*



Clearly sides in a triangle that are longest are across from the largest angles, and sides that are shortest are across from the smallest angles.

Theorem 5-10

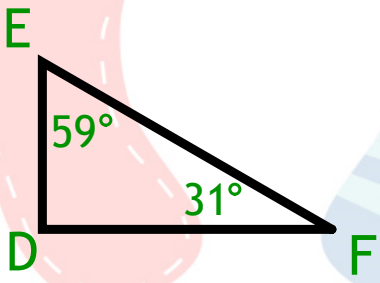
If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.



Because DF > ED, m∠E > m∠F.

Theorem 5-11

If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.

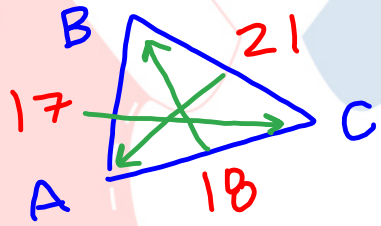


Because $m\angle E > m\angle F$,
$DF > ED$.

Example 1

List the angles of ΔABC from the smallest to the largest.

$AB = 17$, $BC = 21$, $AC = 18$

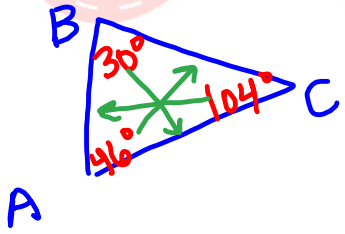


$\angle C, \angle B, \angle A$

Example 2

List the sides of ΔABC from the longest to the shortest.

$m\angle A = 46°$, $m\angle B = 30°$

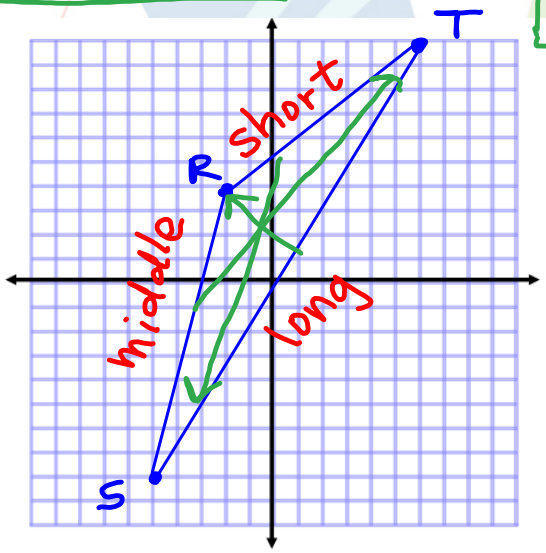


$\overline{AB}, \overline{BC}, \overline{AC}$

Example 3

Draw Triangle RST with vertices R(-2,4), S(-5,-8), and T(6,10). List the angles in order from the greatest measure to the least measure.

$\angle R, \angle T, \angle S$



Example 4

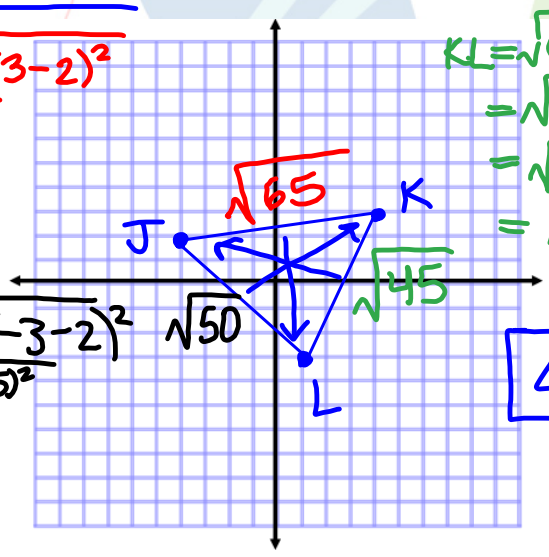
$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Draw Triangle JKL with vertices J(-4,2), K(4,3), and L(1,-3). List the angles in order from the least measure to the greatest measure.

$JK=\sqrt{(4--4)^2+(3-2)^2}$
$=\sqrt{(8)^2+(1)^2}$
$=\sqrt{64+1}$
$=\sqrt{65}$

$JL=\sqrt{(1--4)^2+(-3-2)^2}$
$=\sqrt{(5)^2+(-5)^2}$
$=\sqrt{25+25}$
$=\sqrt{50}$

$KL=\sqrt{(1-4)^2+(-3-3)^2}$
$=\sqrt{(-3)^2+(-6)^2}$
$=\sqrt{9+36}$
$=\sqrt{45}$



$\angle J, \angle K, \angle L$

Example 5

List the sides of ΔABC from shortest to longest if $m\angle A = 9x + 29$, $m\angle B = 93 - 5x$, and $m\angle C = 10x + 2$.

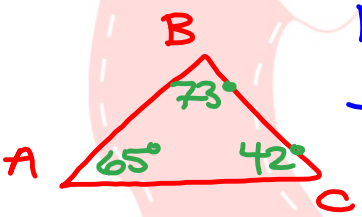
$(9x+29) + (93-5x) + (10x+2) = 180$

$14x + 124 = 180$

$-124 \quad -124$

$\frac{14x}{14} = \frac{56}{14}$

$x = 4$



$m\angle A = 9(4) + 29 = 36 + 29 = 65°$

$m\angle B = 93 - 5(4) = 93 - 20 = 73°$

$m\angle C = 10(4) + 2 = 40 + 2 = 42°$

$\overline{AB}, \overline{BC}, \overline{AC}$

Example 6

List the angles of ΔDEF from largest to smallest if $DE = x + 8$, $EF = 3x - 6$, $DF = 2x + 1$, and the perimeter of the triangle is 57.

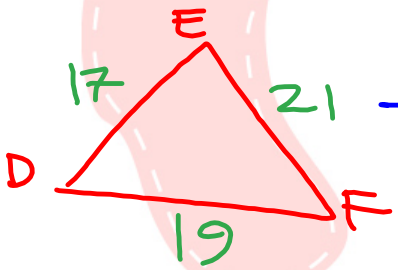
$(x+8) + (3x-6) + (2x+1) = 57$

$6x + 3 = 57$

$-3 \quad -3$

$\frac{6x}{6} = \frac{54}{6}$

$x = 9$



$DE = 9 + 8 = 17$

$EF = 3(9) - 6 = 21$

$DF = 2(9) + 1 = 19$

$\angle D, \angle E, \angle F$