

3.7 Part 2 Graphing a Rational Function

1. Find the asymptotes (vertical, horizontal, or slant).
Graph with dashed lines.
2. Find the x- and y-intercepts, if any.
Plot the points.
3. Use an x-y table to find additional points (3) on each side of the vertical asymptote(s). Plot the points.
4. Sketch the curves through the points.

Example: $y = \frac{x^2 - 4x + 2}{x - 1}$

Holes: none

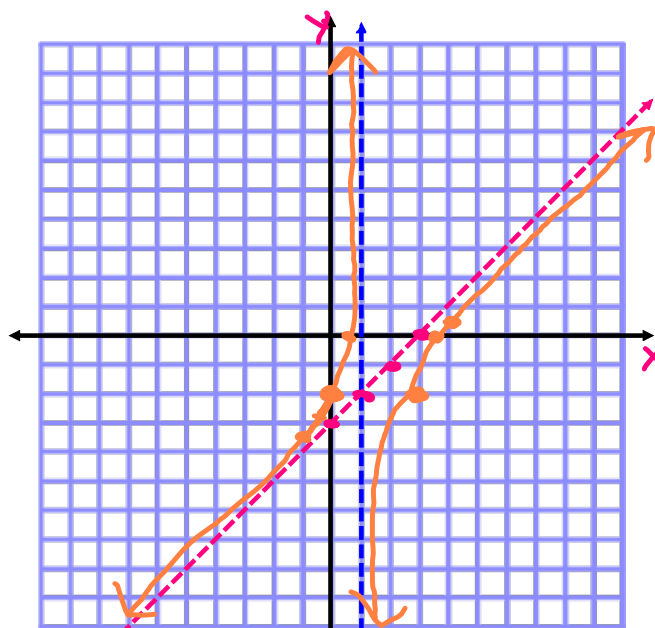
VA: $x = 1$

HA: none

SA: $y = x - 3$

x-int: $2 \pm \sqrt{2}$

y-int: -2



$m=1 \quad b=-3 \rightarrow$
 $2 + \sqrt{2} \quad 2 - \sqrt{2}$

left

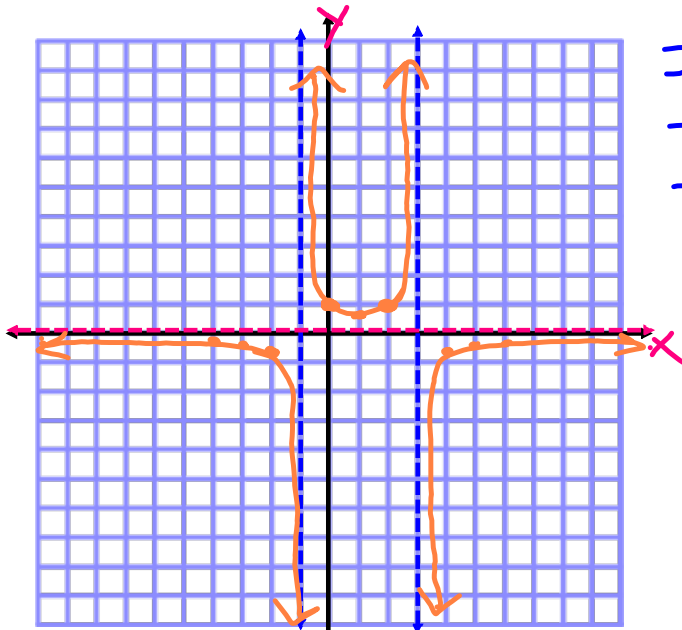
x	y
-1	$\frac{7}{-2}$ or $-3\frac{1}{2}$

right

x	y
2	$\frac{-3}{1} = -2$
4	$\frac{2}{3}$

1	$y = \frac{x^2 - 4x + 2}{x - 1}$	$\frac{x^2 - 4x + 2}{x - 1}$	$(-\infty, 1) \cup (1, \infty)$	$x = 1$	none	none	$y = x - 3$	$x = \frac{2 \pm \sqrt{2}}{2}$	$y = -2$
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Example: $f(x) = \frac{-3}{x^2 - 2x - 3}$



left
 $x \mid y$
 $-2 \mid \frac{-3}{5}$
 $-3 \mid \frac{-3}{2} = -\frac{1}{4}$
 $-4 \mid \frac{-3}{21} = -\frac{1}{7}$

middle
 $x \mid y$
 $1 \mid \frac{-3}{4} = \frac{3}{4}$
 $2 \mid \frac{-3}{-3} = 1$

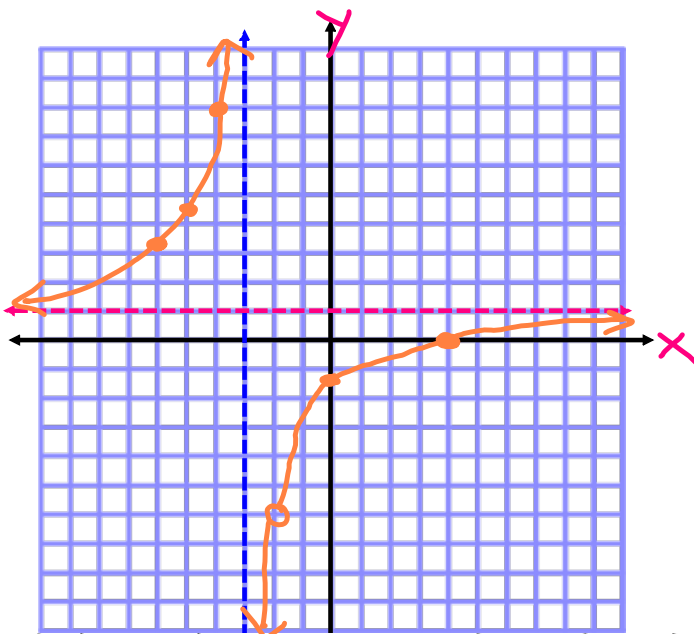
right
 $x \mid y$
 $4 \mid \frac{-3}{5}$
 $5 \mid \frac{-3}{2} = -\frac{1}{4}$
 $6 \mid \frac{-3}{21} = -\frac{1}{7}$

Holes: none
 VA: $x = -1, 3$
 HA: $y = 0$
 SA: none
 x-int: none
 y-int: 1

2	$f(x) = \frac{-3}{x^2 - 2x - 3}$	$\frac{-3}{(x+1)(x-3)}$	$(-\infty, -1) \cup (-1, 3) \cup (3, \infty)$	$x = -1, 3$	none	$y = 0$	none	none	none	$y = 1$
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Example: $y = \frac{x^2 - 2x - 8}{x^2 + 5x + 6}$

$\frac{x-4}{x+3}$



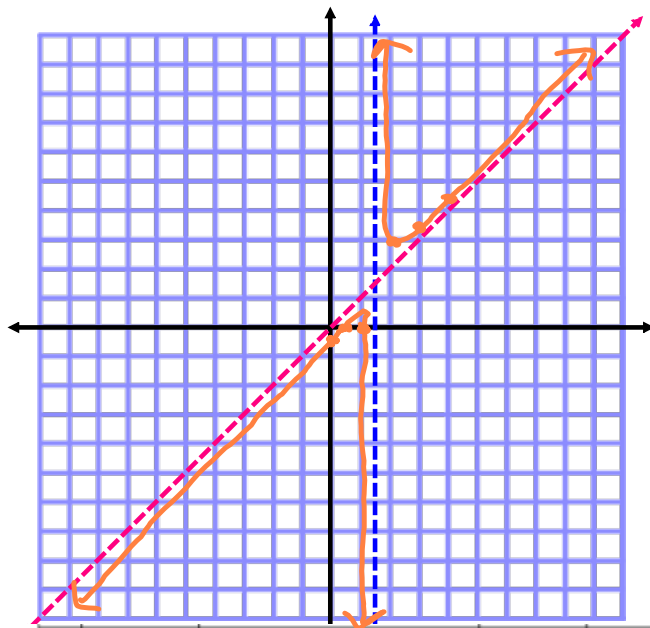
left
 $x \mid y$
 $-4 \mid \frac{0}{-1} = 0$
 $-5 \mid \frac{-9}{-2} = \frac{9}{2}$
 $-6 \mid \frac{-16}{3} = \frac{16}{3}$

Holes: $(-2, -6)$
 VA: $x = -3$
 HA: $y = 1$
 SA: none
 x-int: 4
 y-int: $-4/3$

3	$y = \frac{x^2 - 2x - 8}{x^2 + 5x + 6}$	$\frac{(x-4)(x+2)}{(x+3)(x+2)}$	$(-\infty, -3) \cup (-3, -2) \cup (-2, \infty)$	$x = -3$	$(-2, 6)$	$y = 1$	none	$x = 4$	$y = -4/3$	
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Example: $f(x) = \frac{2x^2 - 3x + 1}{2x - 3} = \frac{(2x-1)(x-1)}{2x-3}$

Holes: none
 VA: $x = 3/2$
 HA: none
 SA: $y = x$
 x-int: $1/2, 1$
 y-int: $-1/3$



$m=1$ $b=0$ ←

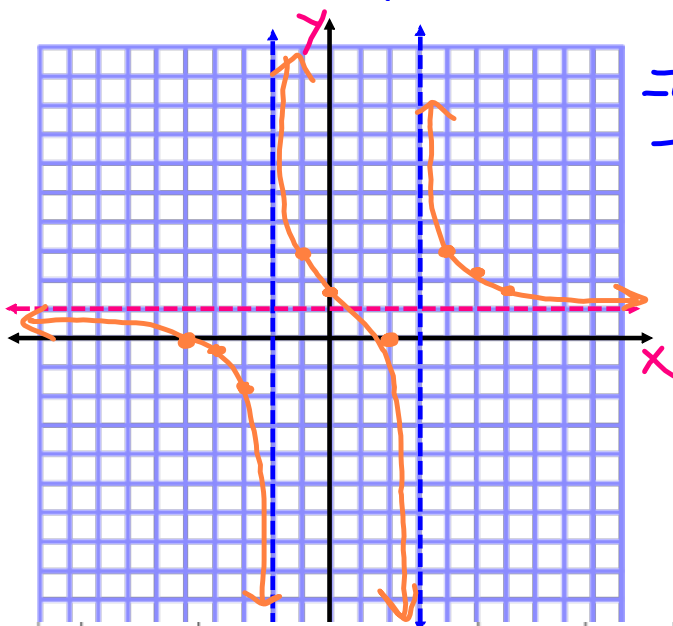
right

x	y
2	$\frac{(3)(1)}{1} = 3$
3	$\frac{(5)(2)}{3} = \frac{10}{3}$
4	$\frac{(7)(3)}{5} = \frac{21}{5}$

4	$f(x) = \frac{2x^2 - 3x + 1}{2x - 3}$	$\frac{(2x-1)(x-1)}{2x-3}$	$(-\infty, 3/2) \cup (3/2, \infty)$	$x = 3/2$	none	none	$y = x$	$x = 1/2, 1$	$y = -1/3$
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Example: $y = \frac{x^2 + 3x - 10}{x^2 - x - 6} = \frac{(x+5)(x-2)}{(x+2)(x-3)}$

Holes: none
 VA: $x = -2, 3$
 HA: $y = 1$
 SA: none
 x-int: $-5, 2$
 y-int: $5/3$



left

x	y
-4	$\frac{-6}{14} = -3/7$
-3	$\frac{-10}{6} = -5/3$

middle

x	y
-1	$\frac{-12}{4} = 3$

right

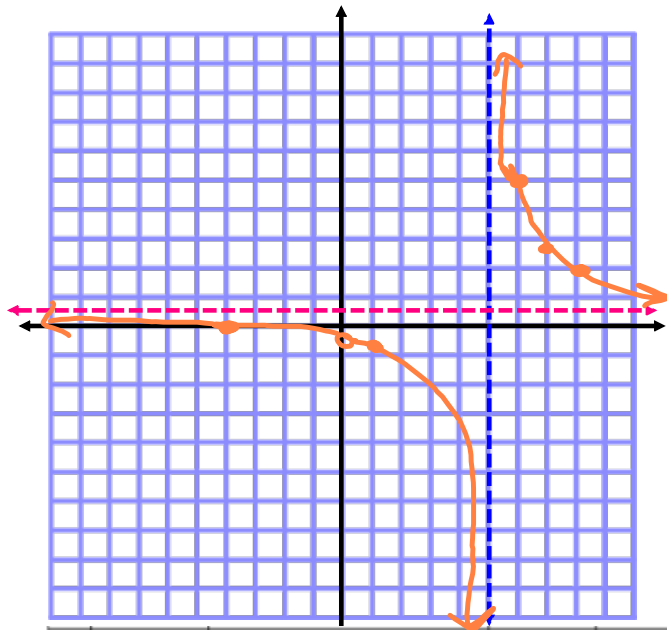
x	y
4	$\frac{18}{6} = 3$
5	$\frac{30}{14} = 15/7$
6	$\frac{42}{24} = 7/4$

5	$y = \frac{x^2 + 3x - 10}{x^2 - x - 6}$	$\frac{(x+5)(x-2)}{(x-3)(x+2)}$	$(-\infty, -2) \cup (-2, 3) \cup (3, \infty)$	$x = -2, 3$	none	$y = 1$	none	$x = -5, 2$	$y = 5/3$
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Example: $f(x) = \frac{x^2 + 4x}{2x^2 - 10x}$

$\frac{x+4}{2(x-5)}$

Holes: (0,-2/5)
 VA: x = 5
 HA: y = 1/2
 SA: none
 x-int: -4
 y-int: none



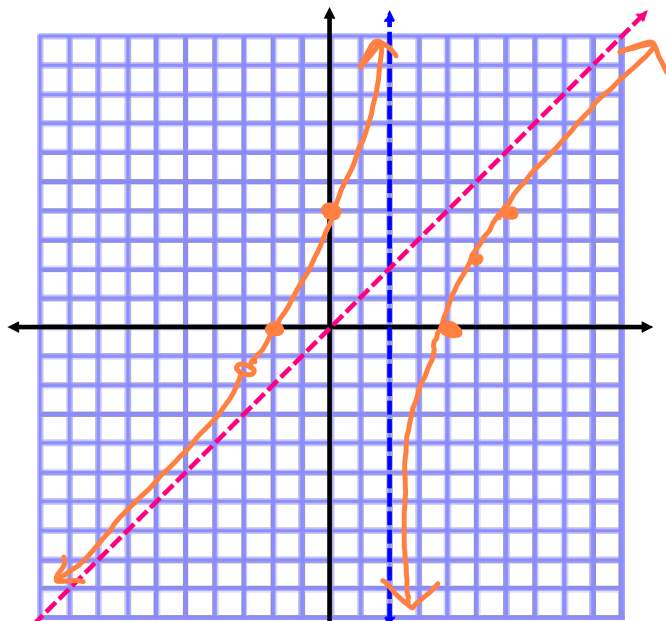
left
 $\frac{x}{y} = \frac{1}{-5/8}$

right
 $\frac{x}{y} = \frac{6}{10/2} = 5$
 $\frac{7}{4}$
 $\frac{8}{6} = 2$

6	$f(x) = \frac{x^2 + 4x}{2x^2 - 10x}$	$\frac{x(x+4)}{2x(x-5)}$	$(-\infty, 0) \cup (0, 5) \cup (5, \infty)$	x = 5	(0, -2/5)	y = 1/2	none	x = -4	none
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Example: $f(x) = \frac{x^3 + x^2 - 14x - 24}{x^2 + x - 6} \cdot \frac{(x-7)(x+2)}{x-2}$

Holes: (-3,-7/5)
 VA: x = 2
 HA: none
 SA: y = x
 x-int: -2, 4
 y-int: 4



right
 $\frac{x}{y} = \frac{5(1)(7)}{3} = \frac{7}{3}$
 $\frac{6}{4} = \frac{16}{4} = 4$

7	$\frac{x^3 + x^2 - 14x - 24}{x^2 + x - 6}$	$\frac{(x+3)(x-4)(x+2)}{(x+3)(x-2)}$	$(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$	x = 2	(-3, -7/5)	none	y = x	x = -2, 4	y = 4
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Attachments

Graph Rational Functions.doc