

7.3 Part 2 DIVIDING POLYNOMIALS

Polynomial Long Division

Example 1Divide $f(x) = x^3 - x^2 + 4x - 10$ by $(x - 2)$.

$$\boxed{x^2 + x + 6 + \frac{2}{x-2}}$$

$$\begin{array}{r} x^2 + x + 6 \\ x - 2 \overline{) x^3 - x^2 + 4x - 10} \\ \underline{+ (-x^3 + 2x^2)} \\ x^2 + 4x \\ \underline{+ (-x^2 + 2x)} \\ 6x - 10 \\ \underline{+ (-6x + 12)} \\ 2 \end{array}$$

Example 2Divide $f(x) = 3x^4 - 5x^3 + 4x - 6$ by $(x + 2)$.

$$\boxed{3x^3 - 11x^2 + 22x - 40 + \frac{74}{x+2}}$$

$$\begin{array}{r} 3x^3 - 11x^2 + 22x - 40 \\ x + 2 \overline{) 3x^4 - 5x^3 + 0x^2 + 4x - 6} \\ \underline{+ (-3x^4 + 6x^3)} \\ -11x^3 + 0x^2 \\ \underline{+ (+11x^3 + 22x^2)} \\ 22x^2 + 4x \\ \underline{+ (-22x^2 + 44x)} \\ -40x - 6 \\ \underline{+ (+40x + 80)} \\ 74 \end{array}$$

Example 3Divide $f(x) = 2x^4 + x^3 + x - 1$ by $(x^2 + 2x - 1)$.

$$\begin{array}{r}
 x^2 + 2x - 1 \overline{) 2x^4 + x^3 + 0x^2 + x - 1} \\
 \underline{-(2x^4 + 4x^3 + 2x^2)} \\
 -3x^3 + 2x^2 + x \\
 \underline{+(3x^3 + 6x^2 + 3x)} \\
 8x^2 - 2x - 1 \\
 \underline{-(8x^2 + 16x + 8)} \\
 -18x + 7
 \end{array}$$

$2x^2 - 3x + 8$
 $2x^2 - 3x + 8 + \frac{-18x + 7}{x^2 + 2x - 1}$

Example 4Divide $f(x) = x^3 + 3x^2 - 7$ by $(x^2 - x - 2)$.

$$\begin{array}{r}
 x^2 - x - 2 \overline{) x^3 + 3x^2 + 0x - 7} \\
 \underline{-(x^3 + x^2 + 2x)} \\
 4x^2 + 2x - 7 \\
 \underline{-(4x^2 + 4x + 8)} \\
 6x + 1
 \end{array}$$

$x + 4$
 $x + 4 + \frac{6x + 1}{x^2 - x - 2}$

Synthetic Division

Synthetic Division is a quick method for dividing polynomials.

BUT it can only be used when the divisor is in the form $x - c$

In synthetic division, we only write the coefficients for the division process.

↑
has to
be linear

Example 5 $2x^3 - x^2 + 0x + 8$ $x + 2 = 0 \rightarrow x = -2 \leftarrow$ Box

Divide $f(x) = 2x^3 - x^2 + 8$ by $(x + 2)$.

$$\begin{array}{r|rrrr}
 -2 & 2 & -1 & 0 & 8 \\
 & \downarrow & -4 & 10 & -20 \\
 \hline
 & 2 & -5 & 10 & -12 \leftarrow \text{remainder}
 \end{array}$$

$$\boxed{2x^2 - 5x + 10 + \frac{-12}{x+2}}$$

Example 6

Divide $f(x) = 2x^4 - 3x^3 - 24x^2 + 13x + 12$ by $(x - 1)$ $x - 1 = 0$

$$\begin{array}{r|rrrrr}
 1 & 2 & -3 & -24 & 13 & 12 \\
 & \downarrow & 2 & -1 & -25 & -12 \\
 \hline
 & 2 & -1 & -25 & -12 & 0 \leftarrow \text{remainder}
 \end{array}$$

$$\boxed{2x^3 - x^2 - 25x - 12}$$

Example 7Divide $f(x) = 9x^4 - 13x^2 + 4$ by $(x + \frac{2}{3})$.

$$\begin{array}{r}
 \underline{-\frac{2}{3}} \Big| \quad 9 \quad 0 \quad -13 \quad 0 \quad 4 \\
 \quad \downarrow \quad -6 \quad 4 \quad 6 \quad -4 \\
 \hline
 9 \quad -6 \quad -9 \quad 6 \quad 0 \leftarrow \text{remainder} \\
 \hline
 \boxed{9x^3 - 6x^2 - 9x + 6}
 \end{array}$$

Example 8 $x^3 + 0x^2 + 0x + 65$ Divide $f(x) = x^3 + 65$ by $(x + 4)$.

$$\begin{array}{r}
 \underline{-4} \Big| \quad 1 \quad 0 \quad 0 \quad 65 \\
 \quad \downarrow \quad -4 \quad 16 \quad -64 \\
 \hline
 1 \quad -4 \quad 16 \quad 1 \leftarrow \text{remainder} \\
 \hline
 \boxed{x^2 - 4x + 16 + \frac{1}{x+4}}
 \end{array}$$