**SPECIAL SEGMENTS IN TRIANGLES REVIEW**

**perpendicular bisector** - a line or line segment that passes through the midpoint of a side of a triangle and is perpendicular to that side

The red lines represent the perpendicular bisectors of each side of the triangle ABC.

Perpendicular bisectors have some special properties.

**Theorem 5-1 Perp. Bisector Theorem**
Any point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment.

**Theorem 5-2 Converse of Perp. Bis. Theorem**
Any point equidistant from the endpoints of a segment lies on the perpendicular bisector of the segment.
An **angle bisector** is a segment that bisects an angle of the triangle and has one endpoint at a vertex of the triangle and the other endpoint at another point on the triangle.

Angle bisectors will always intersect inside the triangle!

**Angle bisectors have some special properties.**

**Theorem 5-3 Angle Bisector Theorem**
Any point on the bisector of an angle is equidistant from the sides of the angle.

**Theorem 5-4 Converse of Angle Bis. Theorem**
Any point on or in the interior of an angle and equidistant from the sides of an angle lies on the bisector of the angle.
5.3 MEDIANS AND ALTITUDES OF TRIANGLES

A **median** is a segment that connects a vertex of a triangle to the midpoint of the opposite side.

Every triangle has 3 medians.

Medians have some special properties.

**Theorem 5-8:** *Concurrency of Medians in a Triangle*

The medians of a triangle intersect at a point that is two-thirds of the distance from each vertex to the midpoint of the opposite side.

- \( P \overline{E} = \frac{1}{2} \overline{AP} \)
- \( 2 \overline{PE} = \overline{AP} \)
- \( \overline{PE} = \frac{1}{3} \overline{AE} \)
- \( \overline{AP} = \frac{2}{3} \overline{AE} \)
- \( \overline{BP} = \frac{2}{3} \overline{BF} \)
- \( \overline{CP} = \frac{2}{3} \overline{CD} \)
EXAMPLE 1

In $\triangle HJK$, P is the centroid & JP = 12. Find PT and JT.

\[
JP = \frac{2}{3} JT
\]

\[
\frac{3}{2} \cdot 12 = \frac{2}{3} JT \cdot \frac{3}{2}
\]

\[
18 = JT
\]

\[
PT = \frac{1}{3} JT
\]

\[
PT = \frac{1}{3} \cdot 18
\]

\[
PT = 6
\]

EXAMPLE 2

In $\triangle ABC$, P is the centroid.

If SC = 2100 ft, find PS & PC.

\[
PC = \frac{2}{3} SC
\]

\[
PC = \frac{2}{3} \cdot 2100
\]

\[
PC = 1400 \text{ ft}
\]

\[
2100 - 1400 = PS
\]

\[
700 \text{ ft} = PS
\]
EXAMPLE 3
In $\triangle ABC$, $P$ is the centroid.
If $BT = 1000$ m, find $TC$ & $BC$.

EXAMPLE 4
In $\triangle ABC$, $P$ is the centroid.
If $PT = 800$ cm, find $PA$ & $TA$.

$PT = \frac{1}{3} \cdot AT$

$3 \cdot 800 = \frac{1}{3} \cdot AT \cdot 3$

$2400 \text{ cm} = AT$

$PA = \frac{2}{3} \cdot AT$

$PA = \frac{2}{3} \cdot 2400$

$PA = 1600 \text{ cm}$
EXAMPLE 5

In $\triangle ABC$, the vertices are A(1,5), B(5,7), and C(9,3). Find the coordinates of the centroid.

- mdpt $AB \left(\frac{1+5}{2}, \frac{5+7}{2}\right) \rightarrow (3,6)$
- mdpt $BC \left(\frac{5+9}{2}, \frac{7+3}{2}\right) \rightarrow (7,5)$
- mdpt $CA \left(\frac{9+1}{2}, \frac{3+5}{2}\right) \rightarrow (5,4)$

EXAMPLE 6

In $\triangle VXW$, the vertices are V(1,1), X(5,2), and W(6,6). Find the coordinates of the centroid.

- mdpt $VX \left(\frac{1+5}{2}, \frac{1+2}{2}\right) \rightarrow (3,1.5)$
- mdpt $XW \left(\frac{5+6}{2}, \frac{2+6}{2}\right) \rightarrow (5.5,4)$
- mdpt $WV \left(\frac{6+1}{2}, \frac{6+1}{2}\right) \rightarrow (3.5,3.5)$
An **altitude** is the perpendicular segment from a vertex of a triangle to the opposite side.

Every triangle has 3 altitudes.

**EXAMPLE 7**

EG is a **median** and an **altitude**. Find $x$ and $y$.

\[
\begin{align*}
11x - 3 &= 6x + 9 + 3 \\
11x &= 6x + 12 \\
-6x &= -6x \\
5x &= \frac{12}{5} \\
x &= \frac{12}{5} \text{ or } 2.4
\end{align*}
\]

\[
\begin{align*}
3(y + 2) &= 90 \\
3y + 6 &= 90 \\
-6 &= -6 \\
3y &= 84 \\
\frac{3y}{3} &= \frac{84}{3} \\
y &= 28
\end{align*}
\]
Triangle ABC has vertices A(-3,10), B(9,2), and C(9,15).

a) Determine the coordinates of point P on AB so that CP is a median of Triangle ABC.

\[
P \left( \frac{-3 + 9}{2}, \frac{10 + 2}{2} \right) = P \left( 3, 6 \right)
\]

b) Determine if CP is an altitude of Triangle ABC.

\[
m_{AB} = \frac{2 - 10}{9 - (-3)} = \frac{-8}{12} = -\frac{2}{3}
\]

\[
m_{CF} = \frac{9 - 6}{9 - 3} = \frac{3}{6} = \frac{1}{2}
\]

Since \( m_{AB} \) and \( m_{CF} \) are slopes of perpendicular lines, CP is an altitude.

Triangle SGB has vertices S(4, 7), G(6, 2), and B(12, -1).

a) Determine the coordinates of point J on GB so that SJ is a median of Triangle SGB.

\[
J \left( \frac{4 + 12}{2}, \frac{7 + (-1)}{2} \right) = J \left( 8, 3 \right)
\]

b) Point M has coordinates (8, 3). Is GM an altitude of Triangle SGB?

\[
m_{SB} = \frac{7 - (-1)}{4 - 12} = \frac{8}{-8} = -1
\]

\[
m_{GM} = \frac{3 - 2}{8 - 6} = \frac{1}{2}
\]

\[
m = \frac{1}{2}
\]

GM is not an altitude because \( m_{GM} \) is not perpendicular to \( m_{SB} \).