7.3 PART 1 FACTORING POLYNOMIALS

I. Sum and Difference of Two Cubes:
a) is a binomial,
b) each term is perfect cube, and
c) terms are connected by addition or subtraction.

Perfect Cubes:

$1=(1)^3$	$216=(6)^3$	$x^3=(x)^3$	$8x^6=(2x^2)^3$
$8=(2)^3$	$343=(7)^3$	$x^6=(x^2)^3$	$216x^{15}=(6x^5)^3$
$27=(3)^3$	$512=(8)^3$	$x^9=(x^3)^3$	$64x^9=(4x^3)^3$
$64=(4)^3$	$729=(9)^3$	$x^{12}=(x^4)^3$	
$125=(5)^3$	$1000=(10)^3$	$x^{15}=(x^5)^3$	

FACTOR PATTERNS

SUM OF 2 CUBES: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ never factors

DIFFERENCE OF 2 CUBES: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Examples:

1. $x^3 - 1000$

$(x)^3$ "a" $(10)^3$ "b"

$(a - b)(a^2 + ab + b^2)$

$(x - 10)(x^2 + x\cdot 10 + 10^2)$

$(x-10)(x^2+10x+100)$

2. $8d^3 + 1$

$(2d)^3$ "a" $(1)^3$ "b"

$(a+b)(a^2-ab+b^2)$

$(2d+1)((2d)^2-2d\cdot 1+1^2)$

$(2d+1)(4d^2-2d+1)$

Examples:

3. $64y^3 - 1$

$(4y)^3 \quad (1)^3$

$(a-b)(a^2+ab+b^2)$

$(4y-1)((4y)^2+4y\cdot 1+1^2)$

$(4y-1)(16y^2+4y+1)$

4. $512 + 27k^3$

$(8)^3 \quad (3k)^3$

$(a+b)(a^2-ab+b^2)$

$(8+3k)(8^2-8\cdot 3k+(3k)^2)$

$(8+3k)(64-24k+9k^2)$

5. $216m^9 + 125$

$(6m^3)^3 \quad (5)^3$

$(a+b)(a^2-ab+b^2)$

$(6m^3+5)((6m^3)^2-6m^3\cdot 5+5^2)$

$(6m^3+5)(36m^6-30m^3+25)$

6. $64 - 343h^3$

$(4)^3 \quad (7h)^3$

$(a-b)(a^2+ab+b^2)$

$(4-7h)(4^2+4\cdot 7h+(7h)^2)$

$(4-7h)(16+28h+49h^2)$

II. GCF Revisited

REMEMBER!! The first step to factoring is LOOK FOR A GCF and factor out!

7. $3d^3 - 81$

$3(d^3-27)$

$(d)^3 \quad (3)^3$

$3(a-b)(a^2+ab+b^2)$

$3(d-3)(d^2+d\cdot 3+3^2)$

$3(d-3)(d^2+3d+9)$

8. $54p^3 + 2$

$2(27p^3+1)$

$(3p)^3 \quad (1)^3$

$2(a+b)(a^2-ab+b^2)$

$2(3p+1)((3p)^2-3p\cdot 1+1^2)$

$2(3p+1)(9p^2-3p+1)$

9. $6w^4 + 48w$

$6w(w^3+8)$

$(w)^3 \quad (2)^3$

$6w(a+b)(a^2-ab+b^2)$

$6w(w+2)(w^2-w\cdot 2+2^2)$

$6w(w+2)(w^2-2w+4)$

10. $16x^5 - 250x^2$

$2x^2(8x^3-125)$

$(2x)^3 \quad (5)^3$

$2x^2(a-b)(a^2+ab+b^2)$

$2x^2(2x-5)((2x)^2+2x\cdot 5+5^2)$

$2x^2(2x-5)(4x^2+10x+25)$

III. Factoring by Grouping

11. $(x^3 - 3x^2) + (5x - 15)$

$x^2(x-3) + 5(x-3)$

$(x-3)(x^2+5)$

12. $(f^3 - f^2) - (9f + 9)$

$f^2(f-1) - 9(f-1)$

$(f-1)(f^2 - 9)$

$(f)^2 \quad (3)^2$

$(f-1)(f-3)(f+3)$

13. $(27q^4 - 27q^3) + (8q - 8)$

$27q^3(q-1) + 8(q-1)$

$(q-1)(27q^3 + 8)$

$(3q)^3 \quad (2)^3$

$(q-1)(a+b)(a^2-ab+b^2)$

$(q-1)(3q+2)((3q)^2 - 3q\cdot 2 + 2^2)$

$(q-1)(3q+2)(9q^2-6q+4)$

14. $(5t^4 - 5t^3) - (20t^2 + 20t)$

$5t^3(t-1) - 20t(t-1)$

$(t-1)(5t^3 - 20t)$

$5t(t-1)(t^2 - 4)$

$(t)^2 \quad (2)^2$

$5t(t-1)(t-2)(t+2)$

IV. Quadratic Techniques

A. Trinomials $ax^n + bx^{\frac{n}{2}} + c$

(exponent of middle term is half the exponent on leading term)

15. $4x^4 - 17x^2 + 4$

sum -17 | product 16

$-4, -4$ | -16 | $\frac{-1}{4}$

$(x^2 - 4)(4x^2 - 1)$

$(x)^2 \quad (2)^2 \quad (2x)^2 \quad (1)$

$(x-2)(x+2)(2x-1)(2x+1)$

16. $2h^4 - 9h^2 + 4$

sum -9 | product 8

$-4, -5$ | $\frac{-1}{2}$

$(h^2 - 4)(2h^2 - 1)$

$(h)^2 \quad (2)^2$

$(h-2)(h+2)(2h^2-1)$

17. $n^4 + 6n^2 + 5$

sum 6 product 5

$5 \quad 1$

$1 \quad 1$

$(n^2 + 5)(n^2 + 1)$

18. $x^4 - 6x^2 - 27$

sum -6 product -27

$-9 \quad 3$

$1 \quad 1$

$(x^2 - 9)(x^2 + 3)$

$(x)^2 \quad (3)^2$

$(x-3)(x+3)(x^2+3)$

B. Binomials (Difference of Squares)

19. $16j^4 - 25$

$(4j^2)^2 \quad (5)^2$

$(4j^2 - 5)(4j^2 + 5)$

20. $2z^5 - 32z$

$2z(z^4 - 16)$

$(z^2)^2 \quad (4)^2$

$2z(z^2 - 4)(z^2 + 4)$

$(z)^2 \quad (2)^2$

$2z(z-2)(z+2)(z^2+4)$

21. $64x^6 - 1$

$(8x^3)^2 \quad (1)^2$

$(8x^3 - 1)(8x^3 + 1)$

$(2x)^3 \quad (1)^3 \quad (2x)^3 \quad (1)^3$

$(2x-1)(4x^2+2x+1)(2x+1)(4x^2-2x+1)$