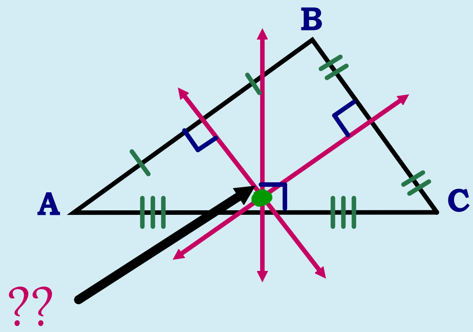
SPECIAL SEGMENTS IN TRIANGLES REVIEW

perpendicular bisector- a line or line segment that passes through the midpoint of a side of a triangle and is perpendicular to that side

The red lines represent the perpendicular bisectors of each side of the triangle ABC.



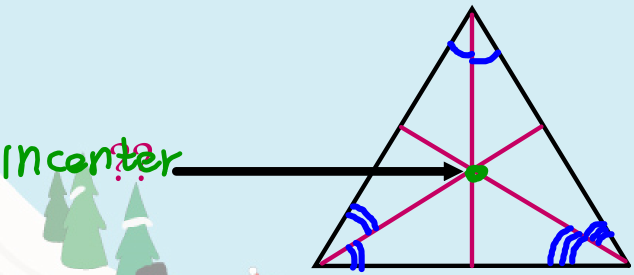
circumcenter

Perpendicular bisectors have some special properties.

Theorem 5-1 Perp. Bisector Theorem
Any point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment.

Theorem 5-2 Converse of Perp. Bis. Theorem
Any point equidistant from the endpoints of a segment lies on the perpendicular bisector of the segment.

An **angle bisector** is a segment that bisects an angle of the triangle and has one endpoint at a vertex of the triangle and the other endpoint at another point on the triangle.



Angle bisectors will always intersect inside the triangle!

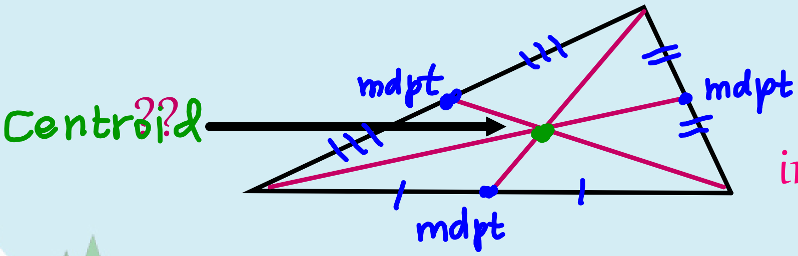
Angle bisectors have some special properties.

Theorem 5-3 Angle Bisector Theorem
Any point on the bisector of an angle is equidistant from the sides of the angle.

Theorem 5-4 Converse of Angle Bis. Theorem
Any point on or in the interior of an angle and equidistant from the sides of an angle lies on the bisector of the angle.

5.4 MEDIANS AND ALTITUDES OF TRIANGLES

A **median** is a segment that connects a vertex of a triangle to the midpoint of the opposite side.



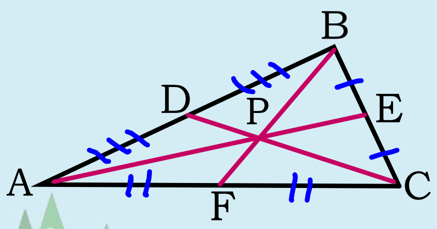
Medians will always intersect inside the triangle!

Every triangle has 3 medians.

Medians have some special properties.

Theorem 5-8: *Concurrency of Medians in a Triangle*

The medians of a triangle intersect at a point that is two-thirds of the distance from each vertex to the midpoint of the opposite side.



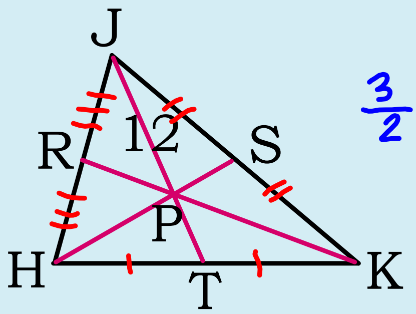
$AP = \frac{2}{3}AE$

$BP = \frac{2}{3}BF$

$CP = \frac{2}{3}CD$

EXAMPLE 1

In $\triangle HJK$, P is the centroid & JP = 12.
Find PT and JT.



$$JP = \frac{2}{3} JT$$

$$\frac{3}{2} \cdot 12 = \frac{2}{3} JT \cdot \frac{3}{2}$$

$$\boxed{18 = JT}$$

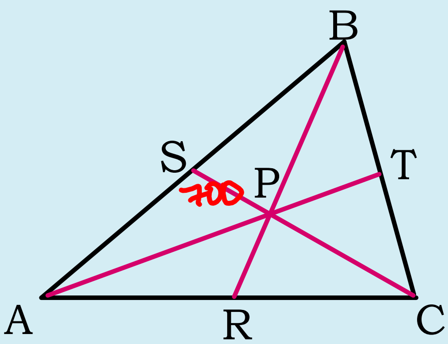
$$JP + PT = JT$$

$$12 + PT = 18$$

$$\boxed{PT = 6}$$

EXAMPLE 2

In $\triangle ABC$, P is the centroid.



If SC = 2100 ft, find PS & PC.

$$PS = \frac{1}{3} SC$$

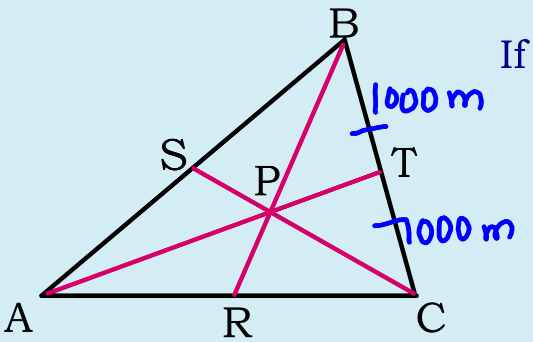
$$PS = \frac{1}{3} \cdot 2100$$

$$\boxed{PS = 700 \text{ ft}}$$

$$\boxed{PC = 1400 \text{ ft}}$$

EXAMPLE 3

In $\triangle ABC$, P is the centroid.

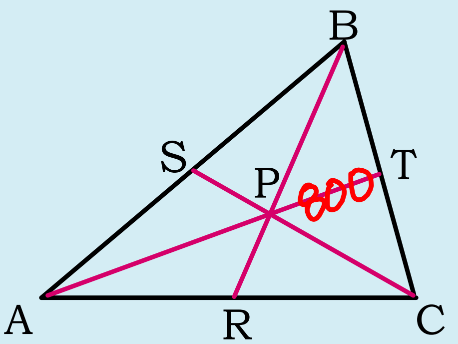


If BT = 1000 m, find TC & BC.

$\boxed{\begin{array}{l} TC = 1000m \\ BC = 2000m \end{array}}$

EXAMPLE 4

In $\triangle ABC$, P is the centroid.



If PT = 800 cm, find PA & TA.

$PA = 2PT$

$PA = 2(800)$

$\boxed{PA = 1600\text{ cm}}$

$\boxed{TA = 2400\text{cm}}$

EXAMPLE 5

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ Midpoint

In $\triangle$ABC, the vertices are A(1,5), B(5,7), and C(9,3). Find the coordinates of the centroid.

AB : $\left(\frac{1+5}{2}, \frac{5+7}{2}\right)$

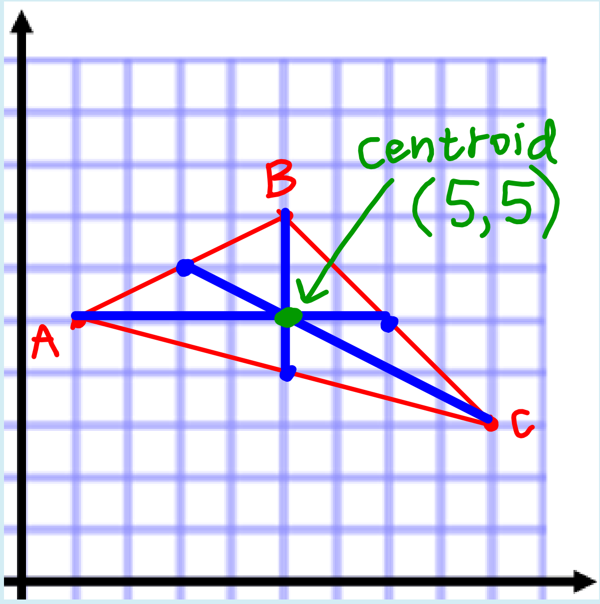
(3, 6) mdpt

BC : $\left(\frac{5+9}{2}, \frac{7+3}{2}\right)$

(7, 5) mdpt

AC : $\left(\frac{1+9}{2}, \frac{5+3}{2}\right)$

(5, 4) mdpt



EXAMPLE 6

In $\triangle$VXW, the vertices are V(1,1), X(5,2), and W(6,6). Find the coordinates of the centroid.

VX: $\left(\frac{1+5}{2}, \frac{1+2}{2}\right)$

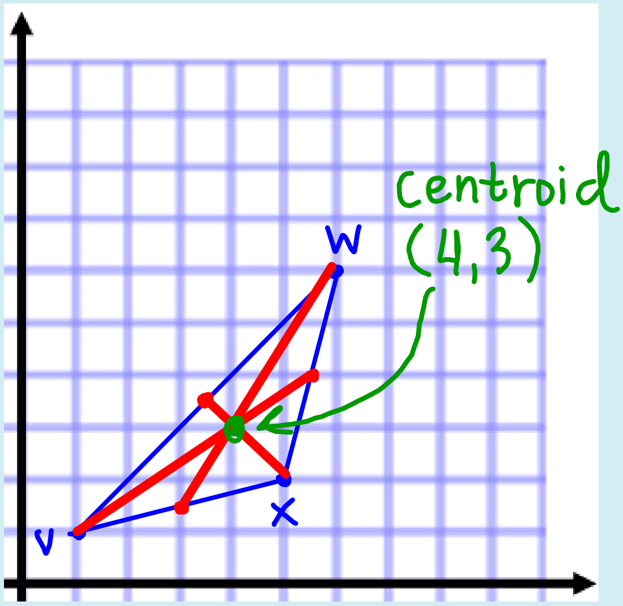
(3, 1.5) mdpt

XW: $\left(\frac{5+6}{2}, \frac{2+6}{2}\right)$

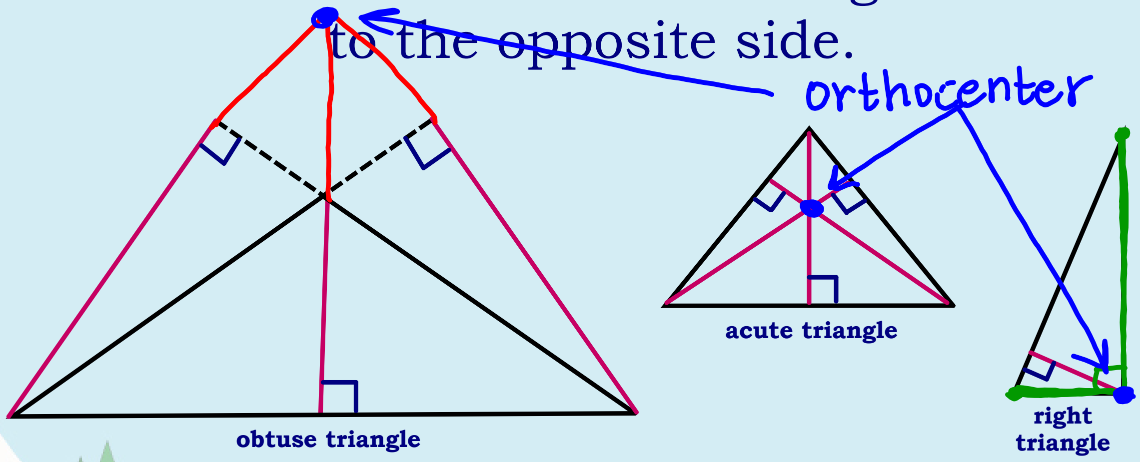
(5.5, 4) mdpt

VW: $\left(\frac{1+6}{2}, \frac{1+6}{2}\right)$

(3.5, 3.5) mdpt



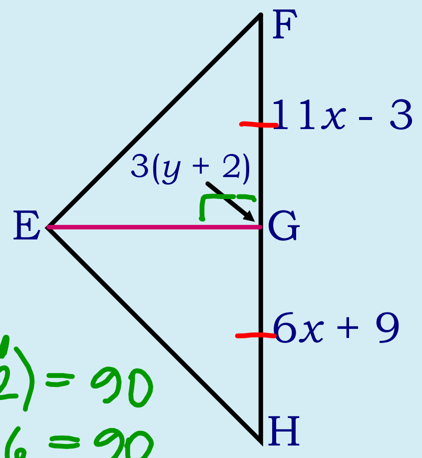
An **altitude** is the perpendicular segment from a vertex of a triangle to the opposite side.



Every triangle has 3 altitudes.

EXAMPLE 7

$\overline{EG}$ is a **median** and an **altitude**. Find x and y.



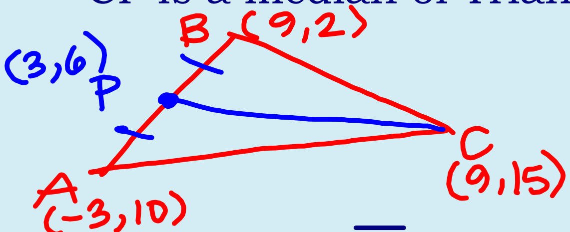
$$11x - 3 = 6x + 9$$
$$-6x \qquad -6x$$
$$5x - 3 = 9$$
$$+3 \qquad +3$$
$$\frac{5x}{5} = \frac{12}{5}$$
$$\boxed{x = \frac{12}{5}}$$

$$3(y+2) = 90$$
$$3y + 6 = 90$$
$$-6 \quad -6$$
$$\frac{3y}{3} = \frac{84}{3}$$
$$\boxed{y = 28}$$

EXAMPLE 8

Triangle ABC has vertices A(-3,10), B(9,2), and C(9,15).

a) Determine the coordinates of point P on $\overline{AB}$ so that $\overline{CP}$ is a median of Triangle ABC.



$\left(\frac{9+-3}{2}, \frac{2+10}{2}\right)$

P (3, 6)

b) Determine if $\overline{CP}$ is an altitude of Triangle ABC.

See if $\overline{AB}$ & $\overline{CP}$ are ⊥ (slope)

form a right ∠

$AB: m = \frac{2-10}{9-(-3)} = \frac{-8}{12} = -\frac{2}{3}$

$CP: m = \frac{15-6}{9-3} = \frac{9}{6} = \frac{3}{2}$

opp. rec. slopes

$\overline{AB} \perp \overline{CP}$

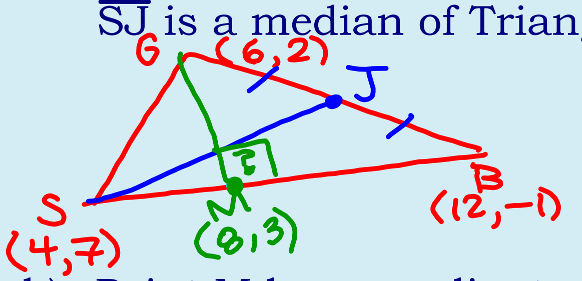
↓

$\overline{CP}$ is an altitude

EXAMPLE 9

Triangle SGB has vertices S(4, 7), G(6, 2), and B(12, -1).

a) Determine the coordinates of point J on $\overline{GB}$ so that $\overline{SJ}$ is a median of Triangle SGB.



$\left(\frac{6+12}{2}, \frac{2+-1}{2}\right)$

$J\left(9, \frac{1}{2}\right)$

b) Point M has coordinates (8, 3). Is $\overline{GM}$ an altitude of Triangle SGB?

$\overline{SB}$ & $\overline{GM}$ ⊥?

$\overline{SB}: m = \frac{-1-7}{12-4} = \frac{-8}{8} = -1$

$\overline{GM}: m = \frac{3-2}{8-6} = \frac{1}{2}$

not opp rec

↓

not ⊥

$\overline{GM}$ is not an altitude