

7.3 PART 1 FACTORING POLYNOMIALS

I. Sum and Difference of Two Cubes:

- a) is a binomial,
- b) each term is perfect cube, and
- c) terms are connected by addition or subtraction.

Perfect Cubes:

$$\begin{array}{llll}
 1 = (1)^3 & 216 = (6)^3 & x^3 = (x)^3 & 8x^6 = (2x^2)^3 \\
 8 = (2)^3 & 343 = (7)^3 & x^6 = (x^2)^3 & 216x^{15} = (6x^5)^3 \\
 27 = (3)^3 & 512 = (8)^3 & x^9 = (x^3)^3 & 64x^9 = (4x^3)^3 \\
 64 = (4)^3 & 729 = (9)^3 & x^{12} = (x^4)^3 & \\
 125 = (5)^3 & 1000 = (10)^3 & x^{15} = (x^5)^3 &
 \end{array}$$

FACTOR PATTERNS

SUM OF 2 CUBES: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
never factors

DIFFERENCE OF 2 CUBES: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Examples:

1. $x^3 - 1000$

$$\begin{array}{ll}
 (x)^3 & (10)^3 \\
 \text{"a"} & \text{"b"}
 \end{array}$$

$$\begin{array}{l}
 (a - b)(a^2 + ab + b^2) \\
 (x - 10)(x^2 + x \cdot 10 + 10^2)
 \end{array}$$

$$\boxed{(x - 10)(x^2 + 10x + 100)}$$

2. $8d^3 + 1$

$$\begin{array}{ll}
 (2d)^3 & (1)^3 \\
 \text{"a"} & \text{"b"}
 \end{array}$$

$$\begin{array}{l}
 (a + b)(a^2 - ab + b^2) \\
 (2d + 1)((2d)^2 - 2d \cdot 1 + 1^2)
 \end{array}$$

$$\boxed{(2d + 1)(4d^2 - 2d + 1)}$$

Examples:

3. $64y^3 - 1$

$$(4y)^3 (1)^3$$

$$(a-b)(a^2+ab+b^2)$$

$$(4y-1)((4y)^2+4y\cdot 1+1^2)$$

$$(4y-1)(16y^2+4y+1)$$

4. $512 + 27k^3$

$$(8)^3 (3k)^3$$

$$(a+b)(a^2-ab+b^2)$$

$$(8+3k)(8^2-8\cdot 3k+(3k)^2)$$

$$(8+3k)(64-24k+9k^2)$$

5. $216m^9 + 125$

$$(6m^3)^3 (5)^3$$

$$(a+b)(a^2-ab+b^2)$$

$$(6m^3+5)((6m^3)^2-6m^3\cdot 5+5^2)$$

$$(6m^3+5)(36m^6-30m^3+25)$$

6. $64 - 343h^3$

$$(4)^3 (7h)^3$$

$$(a-b)(a^2+ab+b^2)$$

$$(4-7h)(4^2+4\cdot 7h+(7h)^2)$$

$$(4-7h)(16+28h+49h^2)$$

II. GCF Revisited

REMEMBER!! The first step to factoring is LOOK FOR A GCF and factor out!

7. $3d^3 - 81$

$$3(d^3 - 27)$$

$$(d)^3 (3)^3$$

$$3(a-b)(a^2+ab+b^2)$$

$$3(d-3)(d^2+d\cdot 3+3^2)$$

$$3(d-3)(d^2+3d+9)$$

8. $54p^3 + 2$

$$2(27p^3 + 1)$$

$$(3p)^3 (1)^3$$

$$2(a+b)(a^2-ab+b^2)$$

$$2(3p+1)((3p)^2-3p\cdot 1+1^2)$$

$$2(3p+1)(9p^2-3p+1)$$

9. $6w^4 + 48w$

$$6w(w^3 + 8)$$

$$(w)^3 (2)^3$$

$$6w(a+b)(a^2-ab+b^2)$$

$$6w(w+2)(w^2-w\cdot 2+2^2)$$

$$6w(w+2)(w^2-2w+4)$$

10. $16x^5 - 250x^2$

$$2x^2(8x^3 - 125)$$

$$(2x)^3 (5)^3$$

$$2x^2(a-b)(a^2+ab+b^2)$$

$$2x^2(2x-5)(2x^2+2x\cdot 5+5^2)$$

$$2x^2(2x-5)(4x^2+10x+25)$$

III. Factoring by Grouping

$$11. (x^3 - 3x^2) + (5x - 15)$$

$$x^2(\cancel{x-3}) + 5(\cancel{x-3})$$

$$(x-3)(x^2 + 5)$$

$$12. (f^3 - f^2)(-9f + 9)$$

$$f^2(\cancel{f-1}) - 9(\cancel{f-1})$$

$$(f-1)(\cancel{f^2} - 9)$$

$$(f-1)(f-3)(f+3)$$

$$13. (27q^4 - 27q^3) + (8q - 8)$$

$$27q^3(\cancel{q-1}) + 8(\cancel{q-1})$$

$$(q-1)(27q^3 + 8)$$

$$(q-1)(3q)^3 + (2)^3$$

$$(q-1)(a+b)(a^2-ab+b^2)$$

$$(q-1)(3q+2)(3q)^2 - 3q \cdot 2 + 2^2$$

$$(q-1)(3q+2)(9q^2 - 6q + 4)$$

$$14. (5t^4 - 5t^3)(20t^2 + 20t)$$

$$5t^3(\cancel{t-1}) - 20t(\cancel{t-1})$$

$$(t-1)(5t^3 - 20t)$$

$$5t(t-1)(\cancel{t^2} - 4)$$

$$5t(t-1)(t-2)(t+2)$$

IV. Quadratic Techniques

A. Trinomials $ax^n + bx^{\frac{n}{2}} + c$

(exponent of middle term is half the exponent on leading term)

$$15. 4x^4 - 17x^2 + 4$$

$$\text{Sum } -17 \mid \text{product } 16$$

$$\frac{-4}{1} \quad \frac{-16}{4} \quad \frac{-1}{4}$$

$$(x^2 - 4)(4x^2 - 1)$$

$$(x-2)(x+2)(2x-1)(2x+1)$$

$$16. 2h^4 - 9h^2 + 4$$

$$\text{sum } -9 \mid \text{product } 8$$

$$\frac{-4}{1} \quad \frac{-9}{2} \quad \frac{-1}{2}$$

$$(h^2 - 4)(2h^2 - 1)$$

$$(h-2)(h+2)(2h^2 - 1)$$

17. $n^4 + 6n^2 + 5$

sum 6 product 5

$$\begin{array}{cc} 5 & 1 \\ \hline & \end{array}$$

$$(n^2 + 5)(n^2 + 1)$$

18. $x^4 - 6x^2 - 27$

sum -6 product -27

$$\begin{array}{cc} -9 & 3 \\ \hline & \end{array}$$

$$(x^2 - 9)(x^2 + 3)$$

$$(x-3)(x+3)(x^2+3)$$

B. Binomials (Difference of Squares)

19. $16j^4 - 25$

$$(4j^2)^2 \quad (5)^2$$

$$(4j^2 - 5)(4j^2 + 5)$$

20. $2z^5 - 32z$

21. $64x^6 - 1$