

## 7.2 GRAPHING POLYNOMIALS

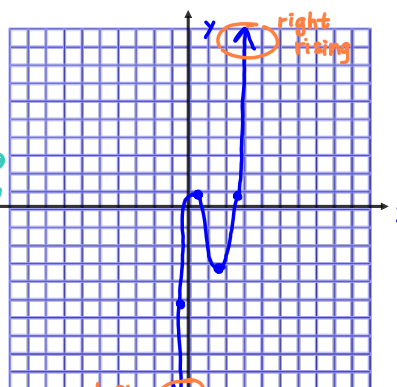
There are several ways to graph polynomial functions. We'll start by making a **table of values**.

Let's use  $x = -3, -2, -1, 0, 1, 2, 3$ .

Graph the function below.

$$f(x) = 3x^3 - 5x^2 - 2x + 1$$

x	y
-3	$3(-3)^3 - 5(-3)^2 - 2(-3) + 1 = -119$
-2	$3(-2)^3 - 5(-2)^2 - 2(-2) + 1 = -39$
-1	$3(-1)^3 - 5(-1)^2 - 2(-1) + 1 = -5$
0	$3(0)^3 - 5(0)^2 - 2(0) + 1 = 1$
1	$3(1)^3 - 5(1)^2 - 2(1) + 1 = -3$
2	$3(2)^3 - 5(2)^2 - 2(2) + 1 = 1$
3	$3(3)^3 - 5(3)^2 - 2(3) + 1 = 31$



What is the shape of the graph?

N-shape

How many U-turns are there?

2

Is the degree even or odd?

odd

Is the leading coefficient pos. or neg.?

positive

Describe the end behavior.

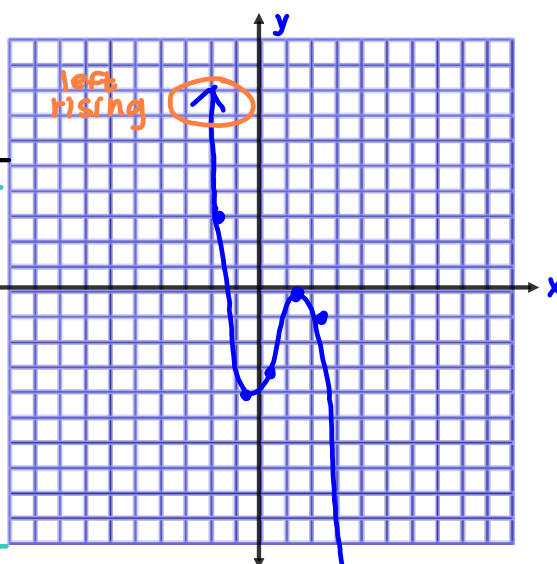
$f(x) \rightarrow -\infty$  as  $x \rightarrow -\infty$   
y-values are falling left

$f(x) \rightarrow \infty$  as  $x \rightarrow \infty$   
y-values are rising right

Graph the function below.

$$f(x) = -x^3 + x^2 + 3x - 3$$

x	y
-3	$-(-3)^3 + (-3)^2 + 3(-3) - 3 = 24$
-2	$-(-2)^3 + (-2)^2 + 3(-2) - 3 = 3$
-1	$-(-1)^3 + (-1)^2 + 3(-1) - 3 = -4$
0	$-(0)^3 + (0)^2 + 3(0) - 3 = -3$
1	$-(1)^3 + (1)^2 + 3(1) - 3 = 0$
2	$-(-2)^3 + (2)^2 + 3(2) - 3 = -1$
3	$-(-3)^3 + (3)^2 + 3(3) - 3 = -12$



What is the shape of the graph?

backwards N

How many U-turns are there?

2

Is the degree even or odd?

odd

Is the leading coefficient pos. or neg.?

negative

Describe the end behavior.

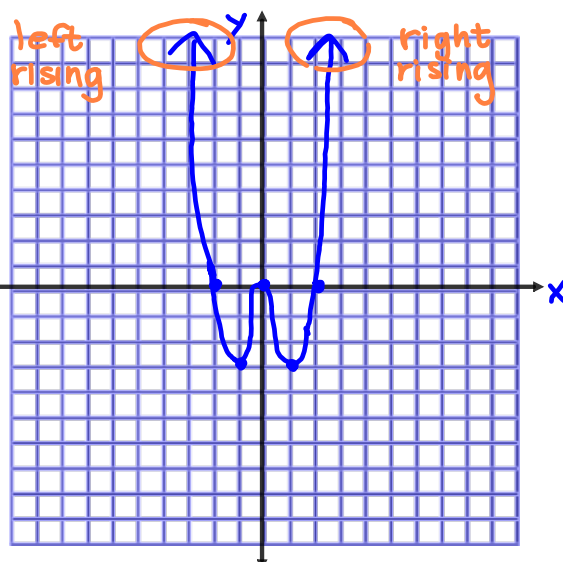
$f(x) \rightarrow \infty$  as  $x \rightarrow -\infty$

$f(x) \rightarrow -\infty$  as  $x \rightarrow \infty$

Graph the function below.

$$f(x) = x^4 - 4x^2$$

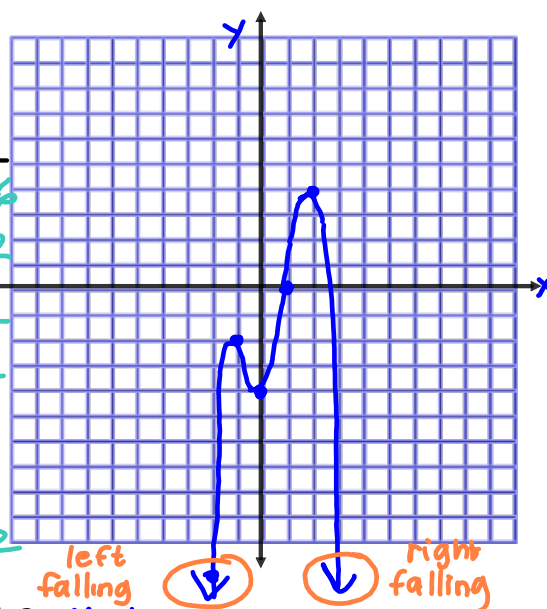
x		y
-3	$(-3)^4 - 4(-3)^2$ <small><math>-81 - 36</math></small>	45
-2	$(-2)^4 - 4(-2)^2$ <small><math>-16 - 16</math></small>	0
-1	$(-1)^4 - 4(-1)^2$ <small><math>-1 - 4</math></small>	3
0	$(0)^4 - 4(0)^2$ <small><math>0 - 0</math></small>	0
1	$(1)^4 - 4(1)^2$ <small><math>1 - 4</math></small>	3
2	$(2)^4 - 4(2)^2$ <small><math>16 - 16</math></small>	0
3	$(3)^4 - 4(3)^2$ <small><math>81 - 36</math></small>	45

What is the shape of the graph? **W-shape**How many U-turns are there? **3**Is the degree even or odd? **even**Is the leading coefficient pos. or neg.? **positive**Describe the end behavior.  $f(x) \rightarrow \infty$  as  $x \rightarrow -\infty$   
 $f(x) \rightarrow \infty$  as  $x \rightarrow \infty$ 

Graph the function below.

$$f(x) = -x^4 + x^3 + 4x^2 - 4$$

x		y
-3	$-(-3)^4 + (-3)^3 + 4(-3)^2 - 4$ <small><math>-81 - 27 + 36 - 4</math></small>	-76
-2	$-(-2)^4 + (-2)^3 + 4(-2)^2 - 4$ <small><math>-16 - 8 + 16 - 4</math></small>	-12
-1	$-(-1)^4 + (-1)^3 + 4(-1)^2 - 4$ <small><math>-1 - 1 + 4 - 4</math></small>	-2
0	$-(0)^4 + (0)^3 + 4(0)^2 - 4$ <small><math>0 + 0 + 0 - 4</math></small>	-4
1	$-(1)^4 + (1)^3 + 4(1)^2 - 4$ <small><math>-1 + 1 + 4 - 4</math></small>	0
2	$-(2)^4 + (2)^3 + 4(2)^2 - 4$ <small><math>-16 + 8 + 16 - 4</math></small>	4
3	$-(3)^4 + (3)^3 + 4(3)^2 - 4$ <small><math>-81 + 27 + 36 - 4</math></small>	-22

What is the shape of the graph? **M-shape**How many U-turns are there? **3**Is the degree even or odd? **even**Is the leading coefficient pos. or neg.? **negative**Describe the end behavior.  $f(x) \rightarrow -\infty$  as  $x \rightarrow -\infty$   
 $f(x) \rightarrow -\infty$  as  $x \rightarrow \infty$

Function	Degree	Name	Sketch of General Shape		Max. Turns	Number of Zeros	End Behavior	
			$a_n > 0$	$a_n < 0$			$a_n > 0$	$a_n < 0$
★ $y = -5$	0	constant			0			
★ $y = 3x + 2$	1	linear			0	1	falling right rising	
★ $y = x^2 + x - 2$	2	quadratic			1	2	left rising right rising	
★ $y = 3x^3 - 12x + 4$	3	cubic			2	3	left falling right rising	
★ $y = x^4 + 2x^3 - 5x^2 - 6x$	4	quartic			3	4	left rising right rising	
★ $y = 6x^5 + 5x^4 - 15x^3 - 10x^2 + 5x + 2$	5	quintic			4	5	left falling right rising	

END BEHAVIOR OF A POLYNOMIAL FUNCTION

	$a$ is positive		$a$ is negative	
	left	right	left	right
degree $n$ is even think parabola	rising	rising	falling	falling
$n$ is odd think LINE	falling	rising	rising	falling

**Practice:** Describe the end behavior of each function below.

1.  $-2x^5 + 3x^2 - x - 5$  *odd deg / LC neg.*  
 $f(x) \rightarrow \infty$  as  $x \rightarrow -\infty$   
 $f(x) \rightarrow -\infty$  as  $x \rightarrow \infty$

2.  $6x^4 + x^3 - 2x^2 - 4x + 1$  *even deg / LC pos.*  
 $f(x) \rightarrow \infty$  as  $x \rightarrow -\infty$   
 $f(x) \rightarrow \infty$  as  $x \rightarrow \infty$

3.  $-7x^6 + 8x^3 - 5$  *even deg / LC neg.*  
 $f(x) \rightarrow -\infty$  as  $x \rightarrow -\infty$   
 $f(x) \rightarrow -\infty$  as  $x \rightarrow \infty$

4.  $5x^3 + x^2 - x - 9$