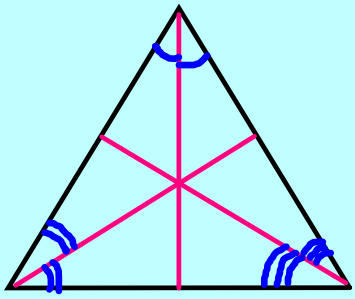
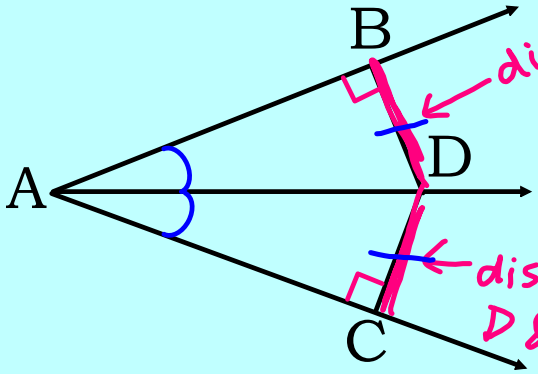
5.3 *Angle Bisectors*

Remember: An angle bisector is a ray that divides an angle into two congruent angles.



Theorem 5.5 ***Angle Bisector Theorem***

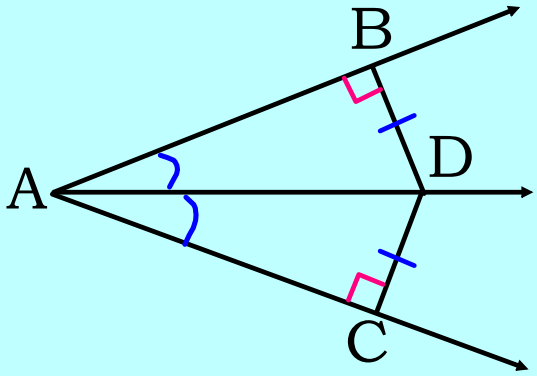
If a point is on the bisector of an angle, then it is equidistant from the two sides of the angle.



Because D is on $\overrightarrow{AD}$, we know that BD = CD.

Theorem 5.6 ***Converse of Angle Bis. Thm***

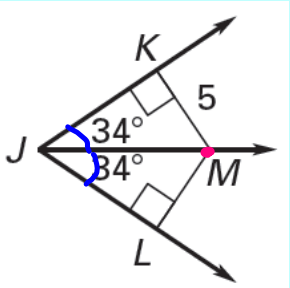
If a point is in the interior of an angle and is equidistant from the sides of the angle, then it lies on the angle bisector of the angle.



Because BD = CD & D is inside ∠BAC, we know that D is on the angle bisector.

***Example* 1**

Find LM.

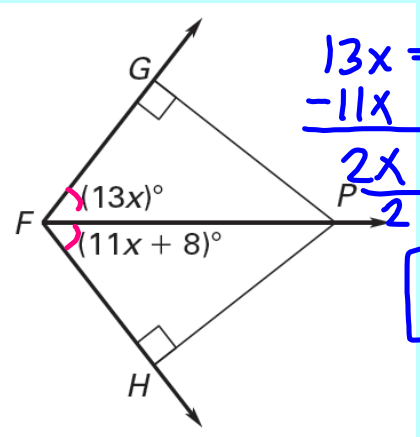


$\overline{JM}$ is ∠ bisector

LM = 5

Example 2

For what value of x does P lie on the angle bisector?



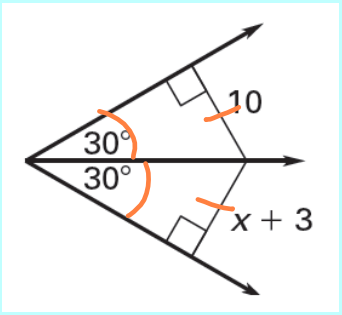
$13x = 11x + 8$

$-11x \quad -11x$

$\frac{2x}{2} = \frac{8}{2}$

$x = 4$

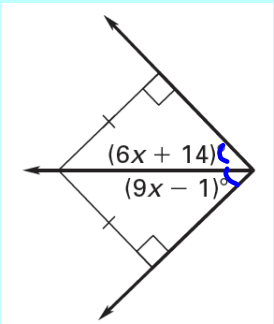
Example 3
Find the value of x.



$x + 3 = 10$

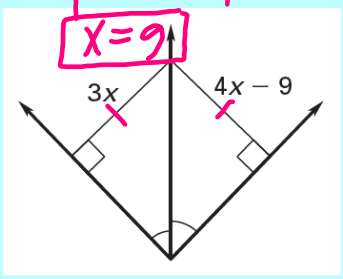
$x = 7$

Example 4
Find the value of x.

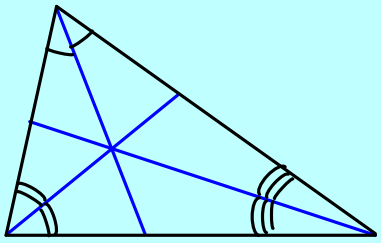


$6x + 14 = 9x - 1$
$-6x \quad -6x$
$14 = 3x - 1$
$+1 \quad +1$
$15 = 3x$
$\frac{15}{3} = \frac{3x}{3}$
$5 = x$

Example 5
Find the value of x.



$3x = 4x - 9$
$-4x \quad -4x$
$-x = -9$
$-1 \quad -1$
$x = 9$



Angle bisectors also produce a point of concurrency.

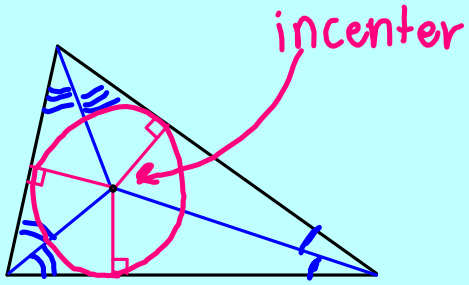
Theorem 5.7

Concurrancy of Angle Bisectors of a Triangle

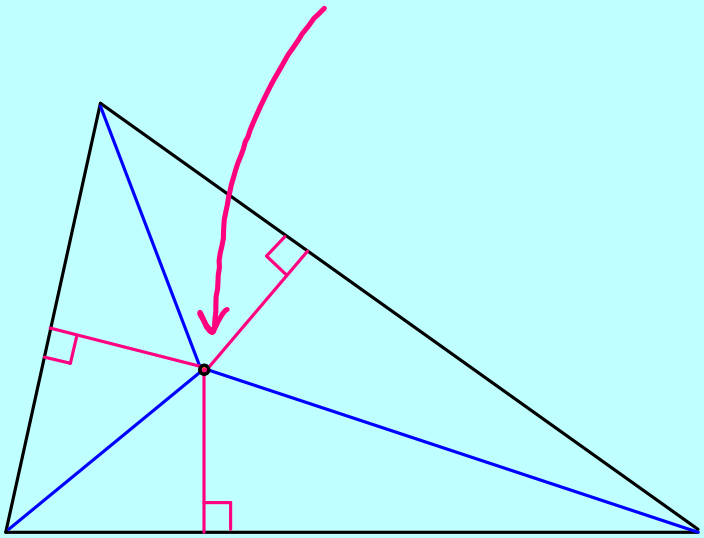
The angle bisectors of a triangle intersect at a point that is equidistant from the sides of the triangle.

blue lines = angle bisectors

pink lines = show the point of concurrency is equidistant from all three sides of the triangle



The point of concurrency of the three angle bisectors of a triangle is called the incenter.



Example 6

The angle bisectors of ΔMNP meet at point L.

a) What segments are congruent?

$LQ = LS = LR$

b) Find LQ and LR. $LQ = 8$ $LR = 8$

$a^2 + b^2 = c^2$

legs hyp.

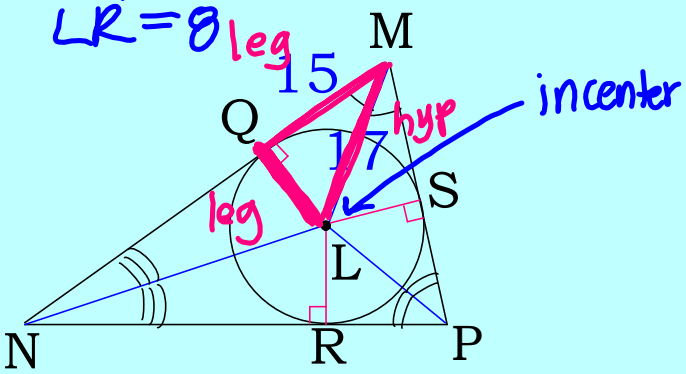
$a^2 + 15^2 = 17^2$

$a^2 + 225 = 289$

$-225 \quad -225$

$\sqrt{a^2} = \sqrt{64}$

$a = 8$

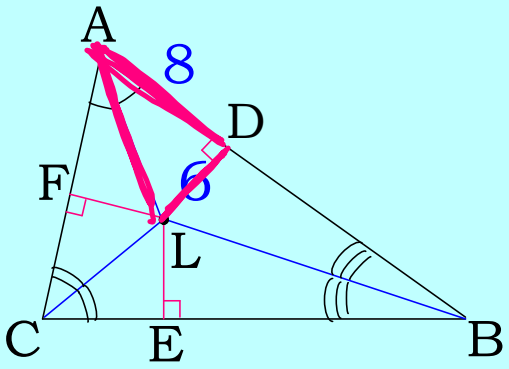


Example 7

The angle bisectors of ΔABC meet at point L.

incenter

Find AL and FL.



$6^2 + 8^2 = c^2$

$36 + 64 = c^2$

$\sqrt{100} = \sqrt{c^2}$

$10 = c$

AL = 10

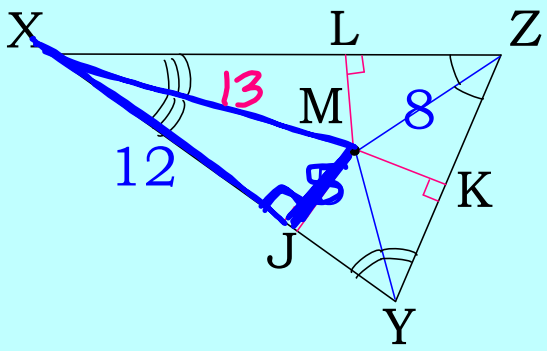
FL = 6

Example 8

The angle bisectors of ΔXYZ meet at point M.

incenter

Find XM and MK.



$5^2 + 12^2 = c^2$

$25 + 144 = c^2$

$\sqrt{169} = \sqrt{c^2}$

$13 = c$

XM = 13

MK = 5