

7.1 An Introduction to Polynomials

monomial - a number, a variable, or a product of numbers and variables

constant - a monomial with no variables

coefficient - the numerical factor in a monomial

degree of a monomial - the sum of the exponents of its variables

$$2x^4$$

deg = 4

$$7xy^2$$

deg = 1 + 2 = 3

$$-5ab^4c^3$$

deg = 1 + 4 + 3 = 8

polynomial - a monomial or a sum of terms that are monomials

binomial - a polynomial with two terms

trinomial - a polynomial with three terms

degree of a polynomial - the same as that of its term with the greatest degree

$$-x^3 + 2x^2 - 5x + 1$$

deg = 3 (circled)
deg = 2
deg = 1
deg = 0
degree = 3

$$-5x^2y^3 + 2xy^7$$

deg = 2 + 3 = 5
deg = 1 + 7 = 8 (circled)
degree = 8

CLASSIFICATION OF POLYNOMIALS BY DEGREE

DEGREE	NAME	EXAMPLE
0	constant	-7
1	linear	$6x + 2$
2	quadratic	$5x^2 - 2x + 9$
3	cubic	$-x^3 + 14$
4	quartic	$8x^4 + 2x - 5$
5	quintic	$x^5 - 3x^4 + 2x^3$

EXAMPLE: Rewrite each polynomial in descending order.

Then classify each polynomial by its degree and number of terms.

1. $2x^2 - 8x - 4x^4$
 $-4x^4 + 2x^2 - 8x$
 quartic trinomial

2. $-9x + 10x^3$
 $10x^3 - 9x$
 cubic binomial

3. $-x^4 + 9x + 7x^5 - 6$
 $7x^5 - x^4 + 9x - 6$
 quintic polynomial

EXAMPLE: Evaluate each polynomial with the given value.

4. Evaluate the polynomial $3x^4 + 2x^2 + 2x - 5$ for $x = -3$.

$$\begin{aligned} & 3(-3)^4 + 2(-3)^2 + 2(-3) - 5 \\ & 243 + 18 - 6 - 5 \\ & 250 \end{aligned}$$

5. Evaluate $\frac{1}{4}x^4 + \frac{1}{8}x^3 + \frac{3}{8}x^2 + \frac{5}{8}x + \frac{7}{8}$ for $x = 2$.

$$\begin{aligned} & \frac{1}{4}(2)^4 + \frac{1}{8}(2)^3 + \frac{3}{8}(2)^2 + \frac{5}{8}(2) + \frac{7}{8} \\ & 4 + 1 + \frac{3}{2} + \frac{5}{4} + \frac{7}{8} \\ & \frac{32}{8} + \frac{8}{8} + \frac{12}{8} + \frac{10}{8} + \frac{7}{8} \\ & \frac{69}{8} \end{aligned}$$

To add and subtract polynomials, combine like terms. Write your answer in standard form, which is with the exponents in **descending** order of degree.

6. Find the sum.

$$\begin{aligned} & (-2x^2 - 3x^3 + 5x + 4) + (-2x^3 + 7x - 6) \\ & -5x^3 - 2x^2 + 12x - 2 \end{aligned}$$

7. Find the difference.

$$\begin{aligned} & (-6x^3 - 6x^2 + 7x - 1) - (3x^3 - 5x^2 - 2x + 8) \\ & (-6x^3 - 6x^2 + 7x - 1) - 3x^3 + 5x^2 + 2x - 8 \\ & -9x^3 - x^2 + 9x - 9 \end{aligned}$$

$$8. (\cancel{2x^4} + 4x^3 + 5x - 2) + (-\cancel{2x^4} - 7x^2 + 8x - 10)$$

$$4x^3 - 7x^2 + 13x - 12$$

$$9. (3x^3 - 12x^2 - 5x + 1) + (\cancel{1}x^2 + \cancel{-5}x + \cancel{-8})$$

$$3x^3 - 11x^2 - 10x - 7$$

$$10. \text{ Multiply } (x - 2)(5x^2 + 3x - 4).$$

$$\times (5x^2 + 3x - 4) - 2(5x^2 + 3x - 4)$$

$$\underline{5x^3 + 3x^2 - 4x} \quad \underline{-10x^2 - 6x + 8}$$

$$5x^3 - 7x^2 - 10x + 8$$

11. Multiply $(x - 1)(x + 2)(x - 3)$.

$$\begin{array}{l}
 \text{FOIL} \\
 (x-1)(x^2 - 3x + 2x - 6) \\
 (x-1)(x^2 - x - 6) \\
 x(x^2 - x - 6) - 1(x^2 - x - 6) \\
 \underline{x^3 - x^2 - 6x} \quad \underline{-x^2 + x + 6} \\
 x^3 - 2x^2 - 5x + 6
 \end{array}$$

12. Multiply $(x + 4)^3$.

$$\begin{array}{l}
 \text{FOIL} \\
 (x+4)(x+4)(x+4) \\
 (x+4)(x^2 + 8x + 16) \\
 x(x^2 + 8x + 16) + 4(x^2 + 8x + 16) \\
 \underline{x^3 + 8x^2 + 16x} \quad \underline{+ 4x^2 + 32x + 64} \\
 x^3 + 12x^2 + 48x + 64
 \end{array}$$

Cube of a Binomial

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

13. Multiply $(4b - 7)^3$.

$$\begin{aligned}
 & a^3 - 3a^2b + 3ab^2 - b^3 \\
 & (4b)^3 - 3(4b)^2(7) + 3(4b)(7)^2 - (7)^3 \\
 & 64b^3 - 3(16b^2)(7) + 3(4b)(49) - 343 \\
 & 64b^3 - 336b^2 + 588b - 343
 \end{aligned}$$

14. Multiply $(pq + 5)^3$.

$$\begin{aligned}
 & a^3 + 3a^2b + 3ab^2 + b^3 \\
 & (pq)^3 + 3(pq)^2(5) + 3(pq)(5)^2 + (5)^3 \\
 & p^3q^3 + 3(p^2q^2)(5) + 3(pq)(25) + 125 \\
 & p^3q^3 + 15p^2q^2 + 75pq + 125
 \end{aligned}$$