5.4 Midsegment Theorem

A **midsegment** of a triangle is a segment that connects the **midpoints** of two sides of the triangle.

**Activity**

Plot the following points: C(-1, 4), D(5, 2), & E(3, 0). Connect to form a triangle.

Find the midpoint of CE and name it Q.

\[
\left(\frac{-1+3}{2}, \frac{4+0}{2}\right) \rightarrow (1, 2) \quad Q
\]

Find the midpoint of CD and name it P.

\[
\left(\frac{-1+5}{2}, \frac{4+2}{2}\right) \rightarrow (2, 3) \quad P
\]

Plot and connect points Q and P.

Find the length of PQ and DE.

\[
PQ = \sqrt{(1-2)^2 + (2-3)^2}
\]

\[
DE = \sqrt{(5-3)^2 + (2-0)^2}
\]

What do you think the relationship is between these lines?

parallel → same slope

midsegment is \(\frac{1}{2}\) of the side it's \(\parallel\) to
Theorem 5.1 **Midsegment Theorem**

The segment connecting the midpoints of two sides of a triangle (midsegment) is parallel to the third side and is half as long as that side.

\[ \text{BE is the } \overline{\text{midsegment}}. \]
\[ \text{BE is parallel to } \overline{\text{CD}}. \]
\[ \text{BE} = \frac{1}{2} \text{CD} \text{ or } \text{CD} = 2 \text{BE}. \]

**Example 1**

**KP** is a midsegment of \( \triangle ABC. \)

Find the value of \( x \).

\[ 25 = 2x \]
\[ 12.5 = x \]

\[ \text{BE} = \frac{1}{2} \text{CD} \text{ or } \text{CD} = 2 \text{BE}. \]
**Example 2**

*KP is a midsegment of $\triangle ABC$. Find the value of $x$.*

$2 \cdot \frac{3}{4} = x$

$\frac{6}{4} = x$

$\frac{3}{2} = x$

**Example 3**

*Fill in the blanks.*

$\overline{WY} \approx \underline{BY} \approx \underline{GU}$

$\overline{YU} \parallel \underline{BM}$

$\overline{GY} \approx \underline{WU} \approx \underline{UM}$

$\overline{GU} \parallel \underline{BW}$
**Example 4**

Use $\triangle ABC$, where $JK$ and $KL$ are midsegments. Find $JK$ and $AB$.

\[ JK = \frac{1}{2} AC \]
\[ JK = \frac{1}{2} \cdot 10 \]
\[ JK = 5 \]

\[ AB = 2KL \]
\[ AB = 2 \cdot 6 \]
\[ AB = 12 \]

**Example 5**

Use $\triangle MNO$, where $X$, $Y$, & $Z$ are midpoints of the sides.

If $YX = 3x - 4$ and $MO = 9x - 20$, find $MO$.

\[ 2xY = MO \quad \iff \quad xY = \frac{1}{2} MO \]
\[ 2(3x - 4) = 9x - 20 \]
\[ 6x - 8 = 9x - 20 \]
\[ 6x = 9x - 12 \]
\[ 3x = 12 \]
\[ x = 4 \]

\[ MO = 9(4) - 20 \]
\[ MO = 36 - 20 \]
\[ MO = 16 \]
**Example 6**

Use \( \triangle MNO \), where \( X, Y, \) & \( Z \) are midpoints of the sides.

If \( YZ = 2x + 3 \)
and \( MN = 5x - 14 \),
find \( YZ \).

\[
2YZ = NM \\
2(2x+3) = 5x-14 \\
4x+6 = 5x-14 \\
20 = x
\]

\( YZ = 2(20) + 3 \)

\( YZ = 43 \)

**Example 7**

Use \( \triangle MNO \), where \( A, B, \) & \( C \) are midpoints of the sides.

If \( AB = 3y - 5 \)
and \( OM = 4y + 2 \),
find \( MC \).

\[
2AB = OM \\
2(3y-5) = 4y+2 \\
6y-10 = 4y+2 \\
2y = 12 \\
y = 6
\]

\( MC = 13 \)