5.1 Perpendicular Bisectors and Angle Bisectors

A perpendicular bisector is a segment, ray, line, or plane that is perpendicular to a segment at its midpoint.

A point is equidistant from two figures if the point is the same distance from each figure.

Point C is equidistant from point A & point B.
Theorem 5.1 **Perpendicular Bisector Theorem**

*If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.*

If $\overline{CD}$ is the perpendicular bisector of $\overline{AB}$, then $CA = CB$.

Theorem 5.2 **Converse of Perp. Bis. Theorem**

*If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.*

If $CA = CB$, then $\overline{CD}$ is the perpendicular bisector of $\overline{AB}$. 
**Example 1**

In the diagram, \( \overline{RS} \) is the perpendicular bisector of \( \overline{PQ} \). Find \( PR \).

\[
\begin{align*}
8x - 9 &= 6x \\
-8x &
\end{align*}
\]

\[
\begin{align*}
-x &= \frac{-2x}{-2} \\
0 &= \frac{9}{2} - x
\end{align*}
\]

\[PR = 8\left(\frac{9}{2}\right) - 9\]

\[PR = 36 - 9\]

\[PR = 27\]

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**Example 2**

In the diagram, \( \overline{JM} \) is the perpendicular bisector of \( \overline{HK} \).

a) Which lengths in the diagram are equal?

- \( LH = LK \)
- \( HJ = JK \)
- \( HM = MK \)

b) Is \( L \) on \( \overline{JM} \)?

Why or why not?

\( L \) is on \( \overline{JM} \) b/c it is equidistant to \( H \) & \( K \).
The **distance from a point to a line** is defined as the length of the perpendicular segment from the point to the line.

Draw the segment that represents the distance from point $P$ to line $m$.

Remember: An **angle bisector** is a ray that divides an angle into two congruent angles.
Theorem 5.3 **Angle Bisector Theorem**

*If a point is on the bisector of an angle, then it is equidistant from the two sides of the angle.*

![Diagram](image)

Because $D$ is on $AD$, we know that $BD = CD$.

Theorem 5.4 **Converse of Angle Bis. Thm**

*If a point is in the interior of an angle and is equidistant from the sides of the angle, then it lies on the angle bisector of the angle.*

![Diagram](image)

Because $BD = CD$ & $D$ is inside $\angle BAC$, we know that $D$ is on the angle bisector.
**Example 3**
Find LM.
\[ LM = 5 \]

**Example 4**
For what value of \( x \) does \( P \) lie on the angle bisector?

\[
\begin{align*}
13x &= 11x + 8 \\
\frac{2x}{2} &= \frac{8}{2}
\end{align*}
\]
\[ x = 4 \]

**Example 5**
Find the value of \( x \).
\[ x + 3 = 10 \\
\[ x = 7 \]

**Example 6**
Find the value of \( x \).

\[
\begin{align*}
6x + 14 &= 9x - 1 \\
6x &= 9x - 1 - 14
\end{align*}
\]
\[ 14 = 3x - 1 + 1 \\
\[ x = \frac{15}{3} \]
\[ x = 5 \]

**Example 7**
Find the value of \( x \).