3.6 Direct Variation

Gloria works for her cousin as her assistant. She earns $9.25 per hour. The table below relates the number of hours that she works (x) and her pay (y).

# of hours	1	2	3	4	5
pay	$9.25	$18.50	$27.75	$37.00	$46.25

Gloria's income depends directly on the number of hours that she works. The relationship between the number of hours worked and her income is shown by the equation $y = 9.25x$.

$y = 9.25x$ is called a direct variation

x & y are going in the same direction

- If x goes up, then y goes up.
- If x goes down, then y goes down.

A direct variation equation is in the form $y = kx$, where $k \neq 0$.

↑ constant of variation

Example 1

Find the constant of variation.

a) $y = 3x$ $k=3$

b) $d = 4t$ $k=4$

c) $\frac{1}{5}m = n$ $k=\frac{1}{5}$

d) $-8c = d$ $k=-8$

Example 2

solve for k

Find the constant of variation.

a) x = 3 when y = 12

$y = kx$

$\frac{12}{3} = \frac{k \cdot 3}{3}$

$4 = k$

b) x = 36 when y = 6

$y = kx$

$\frac{6}{36} \div 6 = \frac{k \cdot 36}{36}$

$\frac{1}{6} = k$

Example 3

$y = kx$

The variables x and y vary directly. Use the given values to write a direct variation equation that relates x and y.

a) x = 2, y = 16

$y = kx$

$\frac{16}{2} = \frac{k \cdot 2}{2}$

$8 = k$

$y = 8x$

b) x = 21, y = 3

$y = kx$

$\frac{3}{21} = \frac{k \cdot 21}{21}$

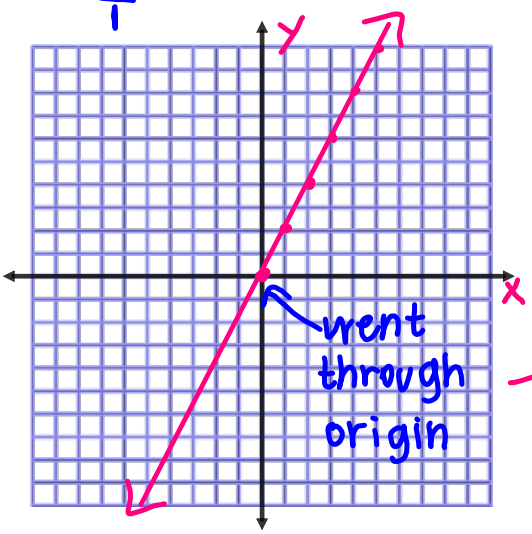
$\frac{1}{7} = k$

$y = \frac{1}{7}x$

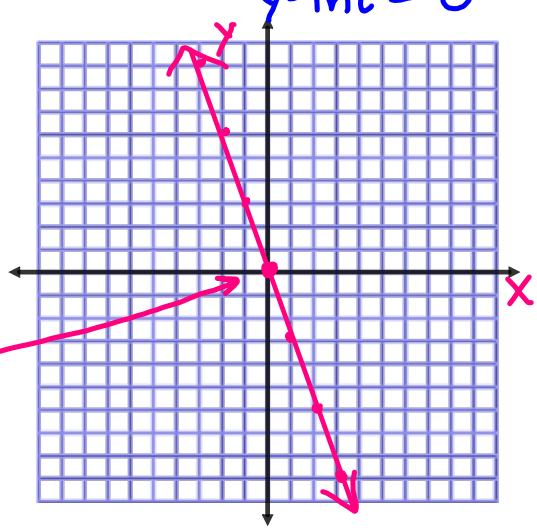
Example 4

Graph each equation.

a) $y = 2x + 0$ — k = slope; y-int = 0



b) $y = -3x$ — $m = -3$; y-int = 0



Example 5

$y = kx$

Tell whether each equation represents direct variation. If so, identify the constant of variation.

a) $-x + y = 1$

$+x \quad +x$

$y = x + 1$

y-int = 1

not direct variation (should have a y-int of zero)

b) $2x + y = 0$

$-2x \quad -2x$

$y = -2x$

or

$y = -2x + 0$

direct variation

$k = -2$

Example 6

If y varies directly as x, and y = 6 when x = 8, find y when x = 12.

$y = kx$

$y = kx$

$\frac{6}{8} = \frac{k \cdot 8}{8}$

$\frac{3}{4} = k$

$y = \frac{3}{4}x$

$y = \frac{3}{4} \cdot 12$

$y = 9$

Example 7

If y varies directly as x, and $y = \frac{1}{4}$ when $x = \frac{1}{8}$, find x when $y = \frac{3}{16}$.

$y = kx$

$\frac{8}{1} \cdot \frac{1}{4} = k \cdot \frac{1}{8} \cdot \frac{8}{1}$

$2 = k$

$y = 2x$

$\frac{1}{2} \cdot \frac{3}{16} = 2x \cdot \frac{1}{2}$

$\frac{3}{32} = x$

Example 8

$y = kx$

$m = k \cdot e$

The weight of an object on the moon varies directly as its weight on Earth. With all his gear on, Neil Armstrong weighed 360 pounds on Earth, but when he stepped on the moon on July 20, 1969, he weighed 60 pounds. Kristina weighs 108 pounds on Earth. What would she weigh on the moon?

$m = k \cdot e$

$\frac{60}{360} = \frac{k \cdot 360}{360}$

$\frac{1}{6} = k$

$m = \frac{1}{6} \cdot e$

$m = \frac{1}{6} \cdot 108$

$m = 18$ pounds

Example 9

varies directly

In an electrical transformer, voltage is directly proportional to the number of turns on the coil. If 110 volts comes from 55 turns, what would be the voltage produced by 66 turns?

$y = kx$

$v = k \cdot c$

$\frac{110}{55} = \frac{k \cdot 55}{55}$

$2 = k$

$v = 2 \cdot c$

$v = 2 \cdot 66$

$v = 132$ volts

Example 10

$y = kx$

A car uses 8 gallons of gasoline (independent/x) to travel 290 miles (dependent/y). How much gasoline will the car use to travel 400 miles?

$y = kx$

$\frac{290}{8} = \frac{k \cdot 8}{8}$

$36.25 = k$

$y = 36.25x$

$\frac{400}{36.25} = \frac{36.25x}{36.25}$

$\frac{320}{29}$ gallons $= x$

Example 11

$y \rightarrow$ cost depends on peanuts $\leftarrow x$

$y = kx$

If 4 pounds of peanuts cost \$7.50,

a) Find the constant of variation.

$\frac{7.50}{4} = \frac{k \cdot 4}{4}$

$\$1.875 = k$

$\$1.88 = k$

b) Write a direct variation equation.

$y = 1.88x$

c) Use your equation to find the cost of 2.5 lb of peanuts?

x

$y = (1.88)(2.5)$

$y = \$4.70$