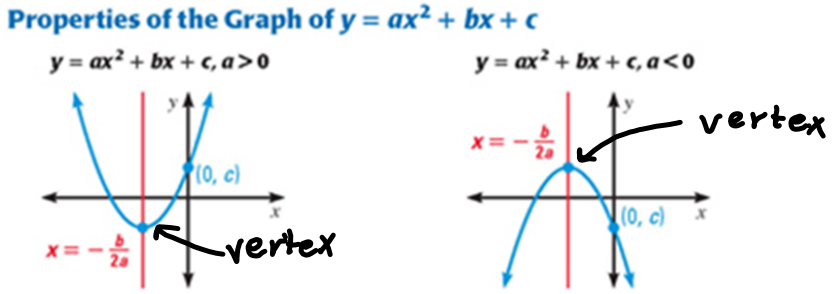
5.1 Part 1 Graphing Quadratic Functions in Standard Form

Standard (Quadratic) Form: $y = ax^2 + bx + c$

A. Identify characteristics



- The graph opens up if $a > 0$ and down if $a < 0$.
- The graph is narrower than the graph of $y = x^2$ if $a > 1$ and wider if $a < 1$.
- The axis of symmetry is $x = -\frac{b}{2a}$ and the vertex has x-coordinate $-\frac{b}{2a}$.

Determine: a) whether the graph opens up or down
b) the axis of symmetry
c) the vertex

$y = ax^2 + bx + c$

$a = -1 \quad b = -2 \quad c = 1$

Example: $f(x) = -x^2 - 2x + 1$

a) opens down b/c a is negative

b) $x = \frac{-b}{2a} \rightarrow x = \frac{2}{2(-1)} \rightarrow x = \frac{2}{-2} \rightarrow x = -1$ a.o.s

c) $f(-1) = -(-1)^2 - 2(-1) + 1 = 2$ vertex $(-1, 2)$

Determine: a) whether the graph opens up or down
b) the axis of symmetry
c) the vertex

$a = \frac{1}{2}$ $b = -3$ $c = 6$

Example: $f(x) = \frac{1}{2}x^2 - 3x + 6$

a) opens up b/c a is positive

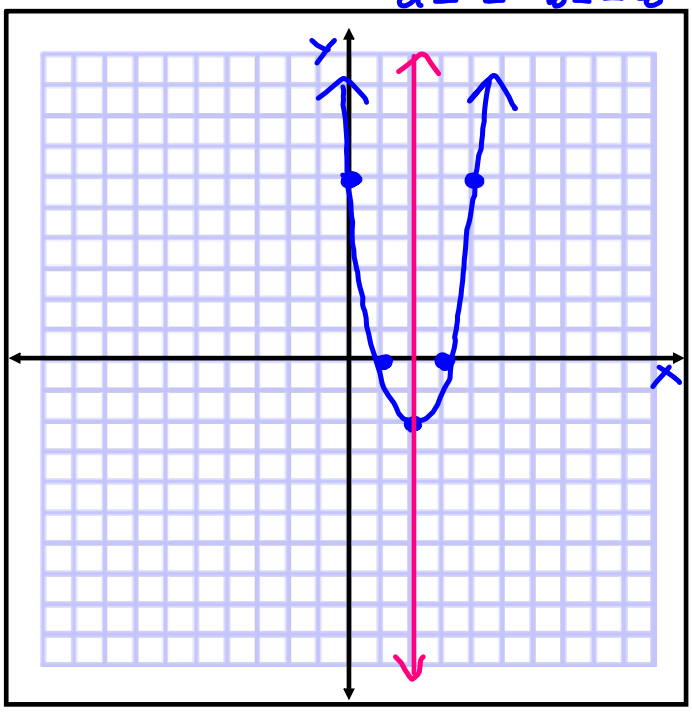
b) $x = \frac{-b}{2a} \rightarrow x = \frac{3}{2(\frac{1}{2})} \rightarrow x = \frac{3}{1} \rightarrow$ $x = 3$ a.o.s.

c) $f(3) = \frac{1}{2}(3)^2 - 3(3) + 6 = \frac{3}{2}$ vertex $(3, \frac{3}{2})$

B. Graph Standard Form

$x = \frac{-b}{2a} = \frac{8}{2(2)} = \frac{8}{4} \rightarrow x = 2$ a.o.s. (vert. line)

Example: $f(x) = 2x^2 - 8x + 6$ $a = 2$ $b = -8$ $c = 6$



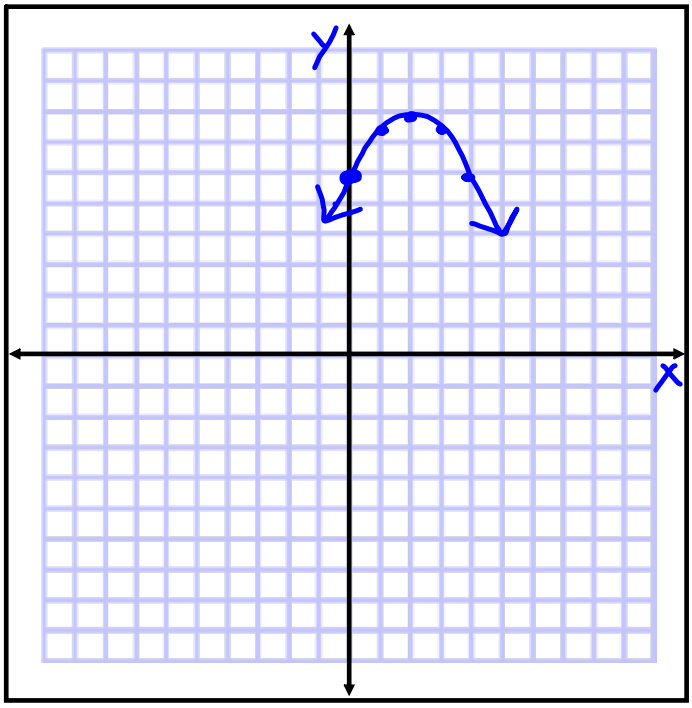
1. Determine if the graph opens up or down.
2. Find the axis of symmetry.
3. Find the vertex & plot.
4. Make a table of values (find two points on either side of the vertex) & plot.
5. Connect the points with a smooth curve.

x		y
0	$2(0)^2 - 8(0) + 6$	6
1	$2(1)^2 - 8(1) + 6$	0
2	$2(2)^2 - 8(2) + 6$	-2
3	$2(3)^2 - 8(3) + 6$	0
4	$2(4)^2 - 8(4) + 6$	6

B. Graph Standard Form

↓ $a=-\frac{1}{2}$ $b=2$ $c=6$ a.o.s. $x=\frac{-b}{2a}$

Example: $g(x) = -\frac{1}{2}x^2 + 2x + 6$ $x=\frac{-2}{2(-\frac{1}{2})} \rightarrow x=2$

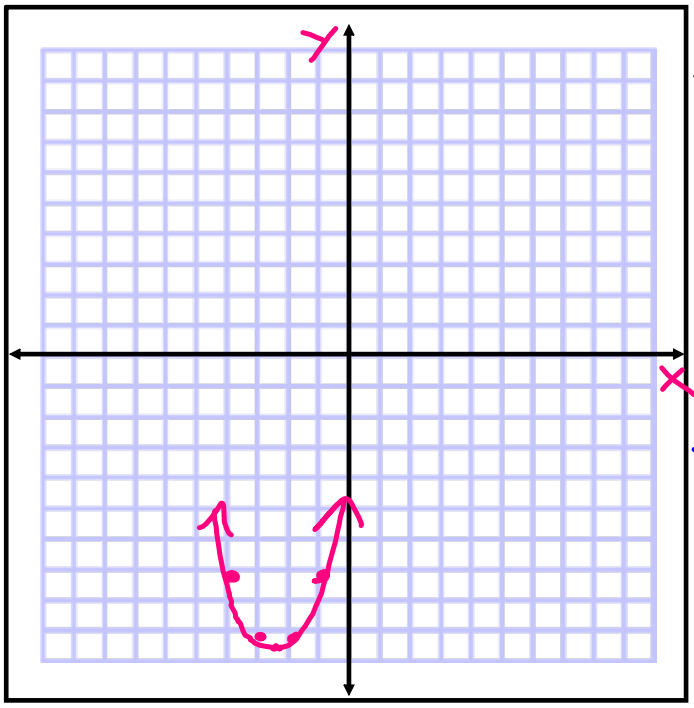


x		y
0	$-\frac{1}{2}(0)^2+2(0)+6$	6
1	$-\frac{1}{2}(1)^2+2(1)+6$	$\frac{15}{2}$ 7.5
2	$-\frac{1}{2}(2)^2+2(2)+6$	8
3	$-\frac{1}{2}(3)^2+2(3)+6$	$\frac{15}{2}$ 7.5
4	$-\frac{1}{2}(4)^2+2(4)+6$	6

B. Graph Standard Form

↑ $a=1$ $b=5$ $c=-3$ $x=\frac{-b}{2a}=\frac{-5}{2(1)} \rightarrow x=-\frac{5}{2}$ or $x=-2.5$

Example: $y = x^2 + 5x - 3$



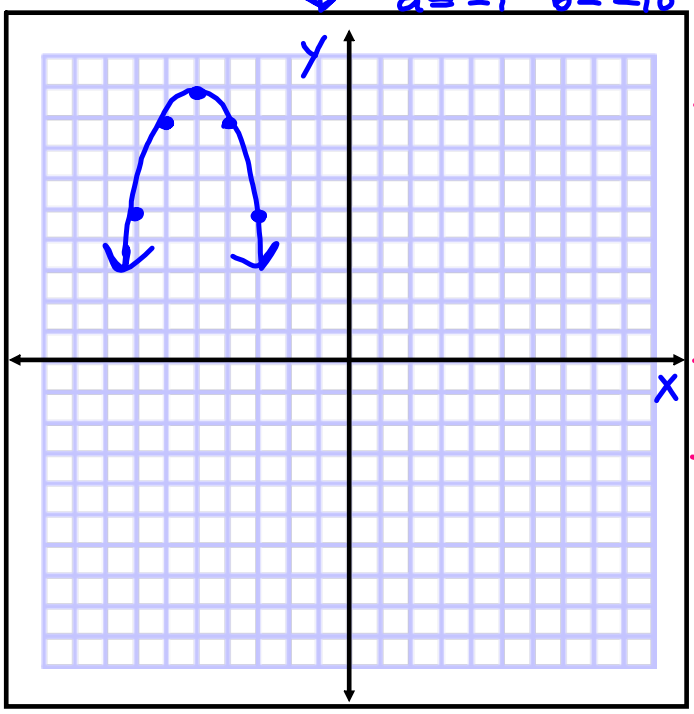
x		y
−1	$(-1)^2+5(-1)-3$	−7
−2	$(-2)^2+5(-2)-3$	−9
−2.5	$(-2.5)^2+5(-2.5)-3$	−9.25
−3	$(-3)^2+5(-3)-3$	−9
−4	$(-4)^2+5(-4)-3$	−7

B. Graph Standard Form

$x = \frac{-b}{2a} = \frac{10}{2(-1)} = \frac{10}{-2} \rightarrow x = -5$ a.o.s.

Example: $f(x) = -x^2 - 10x - 16$

$a = -1$ $b = -10$ $c = -16$



x		y
-3	$-(-3)^2-10(-3)-16$	5
-4	$-(-4)^2-10(-4)-16$	8
-5	$-(-5)^2-10(-5)-16$	9
-6	$-(-6)^2-10(-6)-16$	8
-7	$-(-7)^2-10(-7)-16$	5

C. Find the minimum or maximum value

Example: Tell whether the function $y = 3x^2 - 18x + 20$ has a minimum value or a maximum value. Then find the minimum or maximum value.

opens ↑

y-value of vertex

$$x = \frac{-b}{2a} = \frac{18}{2(3)} = \frac{18}{6} \rightarrow x = 3$$

$y = 3(3)^2 - 18(3) + 20$

$y = -7$

minimum = −7

Example: Follow the same directions with $y = -2x^2 + 4x + 3$.

$a = -2$ $b = 4$ $c = 3$

opens ↓

$$x = \frac{-b}{2a} = \frac{-4}{2(-2)} = \frac{-4}{-4} \rightarrow x = 1$$

$y = -2(1)^2 + 4(1) + 3$

$y = 5$

maximum = 5

Attachments

1-1 Standard Form of Quadratic Functions.doc