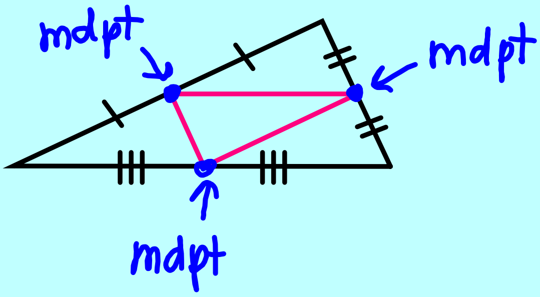
5.1 *Midsegment Theorem*

A midsegment of a triangle is a segment that connects the midpoints of two sides of the triangle.



Activity

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ Mdpt Formula

Plot the following points: C(-1,4), D(5,2), & E(3,0). Connect to form a triangle.

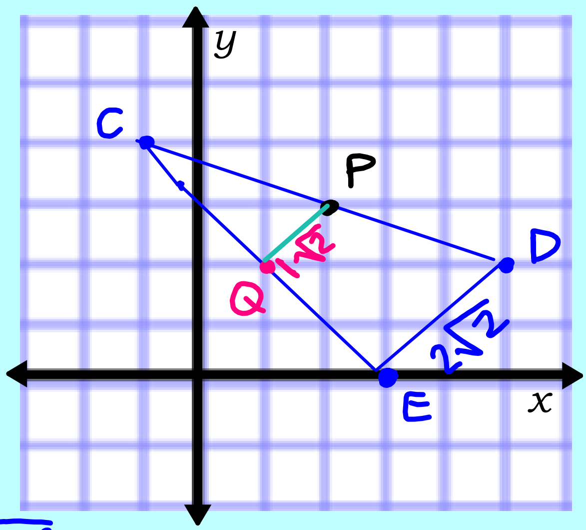
Find the midpoint of CE and name it Q.

$\left(\frac{-1+3}{2}, \frac{4+0}{2}\right) \rightarrow (1,2)$ Q

Find the midpoint of CD and name it P.

$\left(\frac{-1+5}{2}, \frac{4+2}{2}\right) \rightarrow (2,3)$ P

Plot and connect points Q and P.



Find the length of PQ and DE. $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

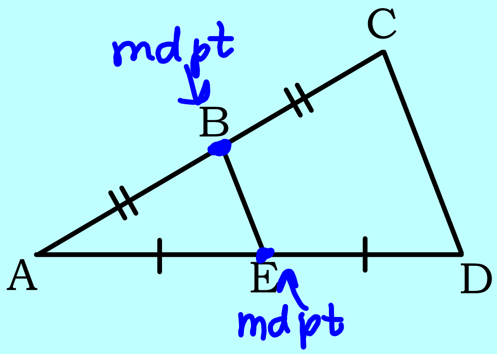
$PQ = \sqrt{(2-1)^2+(3-2)^2}$
$= \sqrt{1+1}$
$= \sqrt{2}$

$DE = \sqrt{(3-5)^2+(0-2)^2}$
$= \sqrt{4+4}$
$= \sqrt{8} = 2\sqrt{2}$

What do you think the relationship is between these lines?

Theorem 5.1 ***Midsegment Theorem***

The segment connecting the midpoints of two sides of a triangle (midsegment) is parallel to the third side and is half as long as that side.

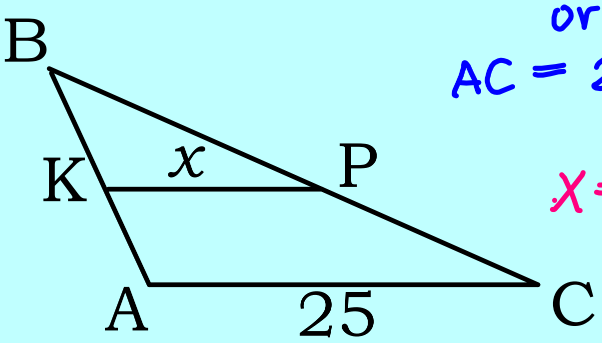


BE is the midsegment.
BE is parallel to CD.
BE = $\frac{1}{2}$ CD or CD = 2 BE.

Example 1

$\overline{KP}$ is a midsegment of $\triangle ABC$.
Find the value of x.

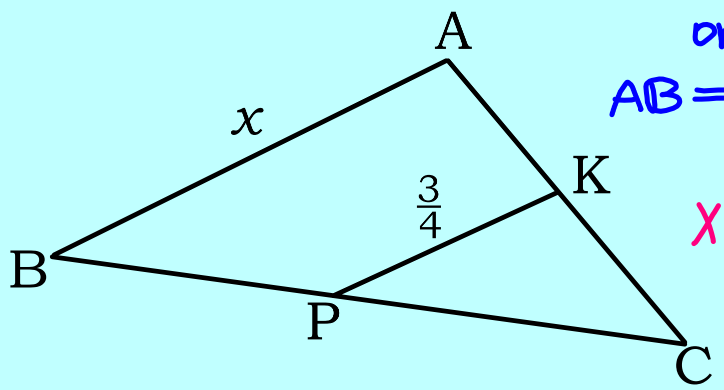
$KP = \frac{1}{2} AC$
or
$AC = 2\ KP$



$x = 12.5$

Example 2

$\overline{KP}$ is a midsegment of $\triangle ABC$.
Find the value of x.



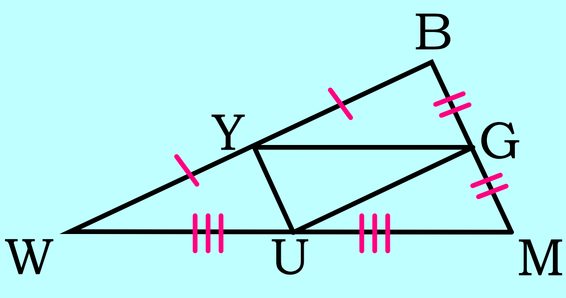
$PK = \frac{1}{2}AB$
or
$AB = 2PK$

$x = \frac{3}{4} \cdot \frac{2}{1}$

$x = \frac{3}{2}$ or 1.5

Example 3

Fill in the blanks.



$\overline{WY} \cong$ $\overline{YB}$ $\cong$ $\overline{UG}$

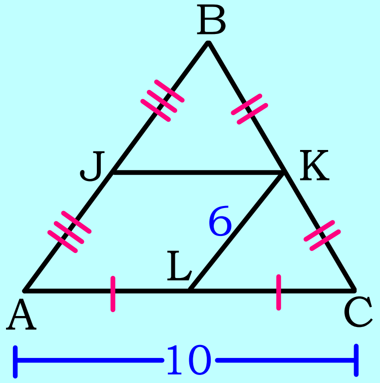
$\overline{YU} \parallel$ $\overline{BM}$

$\overline{GY} \cong$ $\overline{WU}$ $\cong$ $\overline{UM}$

$\overline{GU} \parallel$ $\overline{WB}$

Example 4

Use $\triangle ABC$, where $\overline{JK}$ and $\overline{KL}$ are midsegments. Find JK and AB.



$JK = \frac{1}{2}AC$

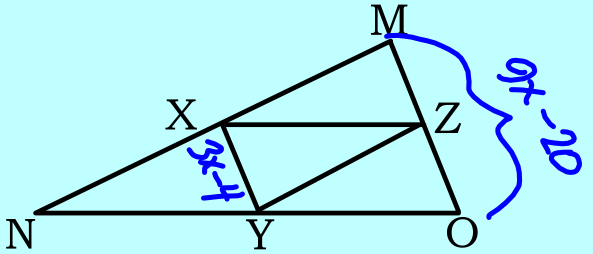
$JK = \frac{1}{2} \cdot 10 = 5$

$AB = 2LK$

$AB = 2 \cdot 6 = 12$

Example 5

Use $\triangle MNO$, where X, Y, & Z are midpoints of the sides.



If $YX = 3x - 4$
and $MO = 9x - 20$,
$\boxed{\text{find } MO.}$

$MO = 2 \cdot XY$

$9x - 20 = 2(3x - 4)$

$9x - 20 = \cancel{6x} - 8$

$\underline{-6x \qquad \cancel{-6x}}$

$3x - \cancel{20} = -8$

$\underline{\cancel{+20} \qquad +20}$

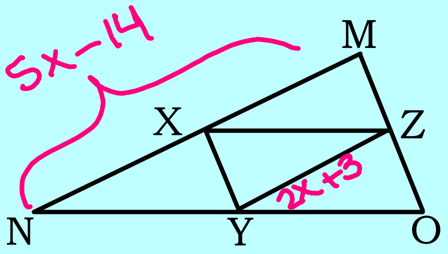
$\frac{3x}{3} = \frac{12}{3}$ $\quad x = 4$

$MO = 9(4) - 20$

$\boxed{MO = 16}$

Example 6

Use △MNO, where X, Y, & Z are midpoints of the sides.



If $YZ = 2x + 3$
and $MN = 5x - 14$,
$\boxed{\text{find } YZ.}$

$YZ = \frac{1}{2} MN$

$2x + 3 = \frac{1}{2}(5x - 14)$

$2x + 3 = \cancel{\frac{5}{2}x} - 7$

$-\frac{5}{2}x \qquad -\cancel{\frac{5}{2}x}$

$-\frac{1}{2}x + \cancel{3} = -7$

$\cancel{-3} \qquad -3$

$-\frac{2}{1} \cdot -\frac{1}{2}x = -10 \cdot -\frac{2}{1}$

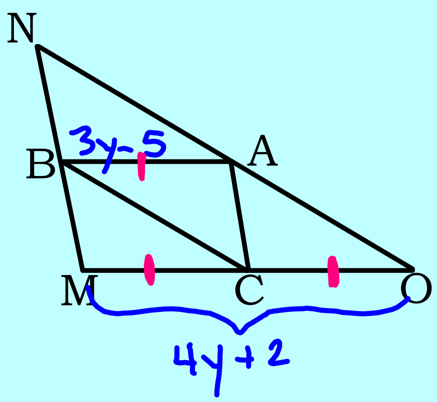
$x = 20$

$YZ = 2(20) + 3$

$\boxed{YZ = 43}$

Example 7

Use △MNO, where A, B, & C are midpoints of the sides.



If $AB = 3y - 5$
and $OM = 4y + 2$,
$\boxed{\text{find } MC.}$

$AB = \frac{1}{2} OM$

$3y - 5 = \frac{1}{2}(4y + 2)$

$3y - 5 = \cancel{2y} + 1$

$-2y \qquad \cancel{-2y}$

$y - 5 = 1$

$+5 \qquad +5$

$y = 6$

$MC = 3(6) - 5$

$\boxed{MC = 13}$