3.3 Part 2 REMAINDER & FACTOR THEOREMS

REMAINDER THEOREM

If the polynomial P(x) is divided by x - c, then the remainder is the value P(c).

Example 1

Use synthetic division and the Remainder Theorem to evaluate P(c) if $P(x) = x^3 - 2x^2 - 5x + 10$ and $c = 1$.

$P(1) = (1)^3 - 2(1)^2 - 5(1) + 10$
$= 1 - 2 - 5 + 10$
$= 4$ rem.

$$\begin{array}{r|rrrr} 1 & 1 & -2 & -5 & 10 \\ & \downarrow & 1 & -1 & -6 \\ \hline & 1 & -1 & -6 & 4 \end{array}$$

rem.

Example 2

Use synthetic division and the Remainder Theorem to evaluate P(c) if $P(x) = x^3 + 2x^2 - 7$ and $c = -2$.

Example 3

Find the remainder when $P(x) = 3x^3 + 4x^2 - 2x + 1$ is divided by $x - \frac{2}{3}$.

$$\begin{array}{r|rrrr} \frac{2}{3} & 3 & 4 & -2 & \cancel{1}\ \frac{3}{3} \\ & \downarrow & 2 & 4 & \frac{4}{3} \\ \hline & 3 & 6 & 2 & \frac{7}{3} \end{array}$$

$$\boxed{\text{remainder} = \frac{7}{3}}$$

FACTOR THEOREM

For a polynomial P(x), x - c is a factor if and only if P(c) = 0.

Example 4

Let $P(x) = x^3 - 7x + 6$. Show that $P(1) = 0$, and use this fact to factor P(x) completely.

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -7 & 6 \\ & \downarrow & 1 & 1 & -6 \\ \hline & 1 & 1 & -6 & 0 \end{array}$$

$x = 1$

rem.

$(x-1)(x^2+x-6)$

$$\boxed{(x-1)(x+3)(x-2) = P(x)}$$

Example 5

Let $P(x) = 2x^3 - 3x^2 - 11x + 6$. Show that $P(-2) = 0$, and use this fact to find all other zeros of $P(x)$.

$$\begin{array}{r|rrrr} -2 & 2 & -3 & -11 & 6 \\ & \downarrow & -4 & 14 & -6 \\ \hline & 2 & -7 & 3 & 0 \leftarrow \text{rem.} \end{array}$$

$(x+2)(2x^2-7x+3)$

$(x+2)(x-3)(2x-1) = P(x)$

$\boxed{\text{zeros} = -2, 3, \tfrac{1}{2}}$

Example 6

Let $P(x) = x^4 - 6x^3 + 3x^2 + 26x - 24$. Show that the given values of c are zeros of $P(x)$, and find all other zeros of $P(x)$.

$c = 3$, $c = -2$

$$\begin{array}{r|rrrrr} 3 & 1 & -6 & 3 & 26 & -24 \\ & \downarrow & 3 & -9 & -18 & 24 \\ \hline -2 & 1 & -3 & -6 & 8 & 0 \leftarrow \text{rem.} \\ & \downarrow & -2 & 10 & -8 & \\ \hline & 1 & -5 & 4 & 0 \leftarrow \text{rem.} & \end{array}$$

$(x-3)(x+2)(x^2-5x+4)$

$(x-3)(x+2)(x-4)(x-1)$

$\boxed{\text{zeros} = 3, -2, 4, 1}$

Example 7

Find a polynomial of degree 4 that has zeros -3, 0, 1, and 5.

$$x(x+3)(x-1)(x-5)$$

$$(x^2+3x)(x^2-6x+5)$$

$$x^2(x^2-6x+5)+3x(x^2-6x+5)$$

$$x^4-\cancel{6x^3}+5x^2+\cancel{3x^3}-18x^2+15x$$

$$\boxed{x^4-3x^3-13x^2+15x}$$

Example 8

Find a polynomial of degree 5 that has zeros -2, -1, 0, 1, and 2.

$$(x+2)(x+1)x(x-1)(x-2)$$

$$x(x+2)(x-2)(x+1)(x-1)$$

$$x(x^2-4)(x^2-1)$$

$$x(x^4-5x^2+4)$$

$$\boxed{x^5-5x^3+4x}$$