

## 3.3 Part 1 DIVIDING POLYNOMIALS

## Polynomial Long Division

Example 1Divide  $f(x) = x^3 - x^2 + 4x - 10$  by  $(x - 2)$ .

$$\begin{array}{r}
 x^2 + x + 6 \\
 \hline
 x - 2 \overline{) x^3 - x^2 + 4x - 10} \\
 \underline{+(x^3 + 2x^2)} \phantom{- 10} \\
 \phantom{x^3} - 3x^2 + 4x \phantom{- 10} \\
 \underline{+(x^2 + 2x)} \phantom{- 10} \\
 \phantom{x^3} \phantom{- 3x^2} 6x - 10 \\
 \underline{+(6x + 12)} \\
 \phantom{x^3} \phantom{- 3x^2} \phantom{6x} - 2
 \end{array}$$

$x^2 + x + 6 + \frac{2}{x-2}$

Example 2Divide  $f(x) = 3x^4 - 5x^3 + 4x - 6$  by  $(x + 2)$ .

$$\begin{array}{r}
 3x^3 - 11x^2 + 22x - 40 \\
 \hline
 x + 2 \overline{) 3x^4 - 5x^3 + 0x^2 + 4x - 6} \\
 \underline{-3x^4 + 6x^3} \phantom{- 6} \\
 \phantom{3x^4} - 11x^3 + 0x^2 \phantom{- 6} \\
 \underline{+11x^3 + 22x^2} \phantom{- 6} \\
 \phantom{3x^4} \phantom{- 11x^3} 22x^2 + 4x \phantom{- 6} \\
 \underline{-22x^2 + 44x} \phantom{- 6} \\
 \phantom{3x^4} \phantom{- 11x^3} \phantom{22x^2} - 40x - 6 \\
 \underline{+40x + 80} \\
 \phantom{3x^4} \phantom{- 11x^3} \phantom{22x^2} \phantom{- 40x} 74
 \end{array}$$

$3x^3 - 11x^2 + 22x - 40 + \frac{74}{x+2}$

Example 3Divide  $f(x) = 2x^4 + x^3 + x - 1$  by  $(x^2 + 2x - 1)$ .

$$\begin{array}{r}
 2x^2 - 3x + 8 \\
 x^2 + 2x - 1 \overline{) 2x^4 + x^3 + 0x^2 + x - 1} \\
 \underline{-2x^4 + 4x^3 + 2x^2} \phantom{-1} \\
 -3x^3 + 2x^2 + x \phantom{-1} \\
 \underline{+3x^3 + 6x^2 + 3x} \phantom{-1} \\
 8x^2 - 2x - 1 \\
 \underline{-8x^2 + 16x + 8} \\
 -18x + 7
 \end{array}$$

$$\boxed{2x^2 - 3x + 8 + \frac{-18x + 7}{x^2 + 2x - 1}}$$

Example 4Divide  $f(x) = x^3 + 3x^2 - 7$  by  $(x^2 - x - 2)$ .

$$\begin{array}{r}
 x + 4 \\
 x^2 - x - 2 \overline{) x^3 + 3x^2 + 0x - 7} \\
 \underline{-x^3 + x^2 + 2x} \phantom{-7} \\
 4x^2 + 2x - 7 \\
 \underline{-4x^2 + 4x + 8} \\
 6x + 1
 \end{array}$$

$$\boxed{x + 4 + \frac{6x + 1}{x^2 - x - 2}}$$

## Synthetic Division

Synthetic Division is a quick method for dividing polynomials.

BUT it can only be used when the divisor is in the form  $x - c$

In synthetic division, we only write the coefficients for the division process.

### Example 5

Divide  $f(x) = 2x^3 - x^2 + 8$  by  $(x + 2)$ .

$$\begin{array}{r} x+2=0 \\ -2 \quad -2 \\ \hline x=-2 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 2 & -1 & 0 & 8 \\ & \downarrow & -4 & 10 & -20 \\ \hline & 2 & -5 & 10 & -12 \end{array} \leftarrow \text{remainder}$$

$$\frac{2x^2 - 5x + 10 + \frac{-12}{x+2}}{x+2}$$

### Example 6

Divide  $f(x) = 2x^4 - 3x^3 - 24x^2 + 13x + 12$  by  $(x - 1)$ .

$$\begin{array}{r} x-1=0 \\ +1 \quad +1 \\ \hline x=1 \end{array}$$

$$\begin{array}{r|rrrrrr} 1 & 2 & -3 & -24 & 13 & 12 \\ & \downarrow & 2 & -1 & -25 & -12 \\ \hline & 2 & -1 & -25 & -12 & 0 \end{array} \leftarrow \text{rem.}$$

$$2x^3 - x^2 - 25x - 12$$

Example 7Divide  $f(x) = 9x^4 - 13x^2 + 4$  by  $(x + \frac{2}{3})$ .

$$\begin{array}{r}
 \underline{-\frac{2}{3}} \Big| \quad 9 \quad 0 \quad -13 \quad 0 \quad 4 \\
 \quad \downarrow \quad -6 \quad 4 \quad 6 \quad -4 \\
 \hline
 9 \quad -6 \quad -9 \quad 6 \quad 0 \quad \leftarrow \text{Rem.}
 \end{array}$$

$$\boxed{9x^3 - 6x^2 - 9x + 6}$$

Example 8Divide  $f(x) = x^4 - 3x^3 - 11x^2 + 8x - 11$  by  $(x - 5)$ .

$$\begin{array}{r}
 \underline{5} \Big| \quad 1 \quad -3 \quad -11 \quad 8 \quad -11 \\
 \quad \downarrow \quad 5 \quad 10 \quad -5 \quad 15 \\
 \hline
 1 \quad 2 \quad -1 \quad 3 \quad 4
 \end{array}$$

$$\boxed{x^3 + 2x^2 - x + 3 + \frac{4}{x-5}}$$