CHAPTER 5 EXTENSION
APPLICATIONS OF QUADRATIC FUNCTIONS

Vertical Motion Problems

Dropped Object Function (on Earth)

$$h(t) = -16t^2 + h_0$$

- t is the time the object has been traveling
- h_0 is the original or initial height of the object

Types of Questions You May Be Asked

- When will the object hit the ground?
- How high is the object at _____ seconds?
- When will the object reach _____ feet?

EXAMPLE 1

A coyote is standing on a cliff 254 feet (h_0) above a roadrunner. If the coyote drops a boulder from the cliff, how much time does the roadrunner have to move out of its way? $t = ?$

$$h(t) = -16t^2 + h_0$$

$$0 = -16t^2 + \cancel{254}$$

$$-254 \qquad\qquad -\cancel{254}$$

$$\frac{-254}{-16} = \frac{-\cancel{16}t^2}{-\cancel{16}}$$

$$\sqrt{15.875} = \sqrt{t^2}$$

$$\boxed{3.98\text{sec} \approx t}$$

EXAMPLE 2

An apple falls from the branch of a tree 30 feet above a man sitting underneath. When will the apple strike the man's head if his head is 2.5 feet above the ground?

h_0

$t = ?$

$h(t)$

$$h(t) = -16t^2 + h_0$$

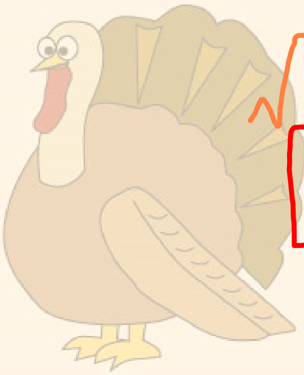
$$2.5 = -16t^2 + 3\cancel{0}$$

$$-30 \qquad\qquad -\cancel{30}$$

$$\frac{-27.5}{-16} = \frac{-16t^2}{-16}$$

$$\sqrt{1.71875} = \sqrt{t^2}$$

$$\boxed{1.31 \text{ sec} \approx t}$$



Launched or Thrown Object Function (on Earth)

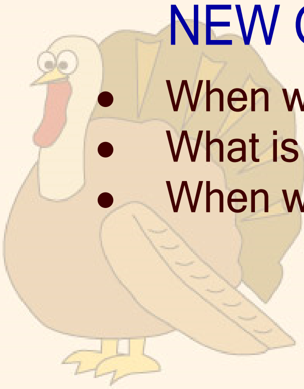
$$h(t) = -16t^2 + v_0t + h_0$$

- t is the time the object has been traveling
- h_0 is the original or initial height of the object
- v_0 is the original or initial velocity of the object

If the object is propelled downward, v_0 is negative.
If the object is propelled upward, v_0 is positive.

NEW Questions You May Be Asked

- When will the object reach its maximum height?
- What is the object's maximum height?
- When will the object reach the initial height again?



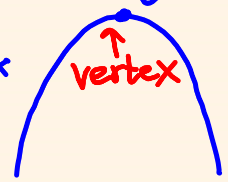
EXAMPLE 3

An arrow is shot upward from a height of 5 feet at an initial velocity of 56 feet per second. v_0 h_0

a) How long after the arrow is released does it reach its maximum height? $t = ?$ at vertex

max height

b) What is the maximum height? y-value of vertex



$$h(t) = -16t^2 + v_0 t + h_0$$

a) $h(t) = -16t^2 + 56t + 5$

a.o.s. $t = \frac{-b}{2a}$ $\quad t = \frac{-56}{2(-16)} = \frac{-56}{-32}$ $\quad \boxed{t = 1.75 \text{ sec}}$

b) $h(1.75) = -16(1.75)^2 + 56(1.75) + 5$

$h(1.75) = 54$ feet $\quad \boxed{\text{max height} = 54 \text{ ft}}$

EXAMPLE 4

Travis hits a foul ball straight up over home plate from a height of 3.5 feet with an initial velocity of 48 feet per second. How long does the catcher have to get ready to catch the ball if he catches the ball 1 foot above the ground?

h_0 $\quad v_0$ $\quad t = ?$ $\quad h(t)$

$a = -16$
$b = 48$
$c = 2.5$

$$h(t) = -16t^2 + v_0 t + h_0$$

$$1 = -16t^2 + 48t + 3.5$$
$$-1 \qquad\qquad\qquad\qquad -1$$
$$0 = -16t^2 + 48t + 2.5$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-48 \pm \sqrt{(48)^2 - 4(-16)(2.5)}}{2(-16)}$$

$t \approx \cancel{-.05}$ $\quad \boxed{t \approx 3.05 \text{ s}}$

EXAMPLE 5

An object is propelled upward from the top of a $\boxed{\text{500 foot}}$ building at an initial velocity of $\boxed{\text{100 feet per second.}}$

h_0 V_0

a) What is the maximum height the object reaches? y-value of vertex

b) How long is the object in the air? t = ? when h(t) = 0

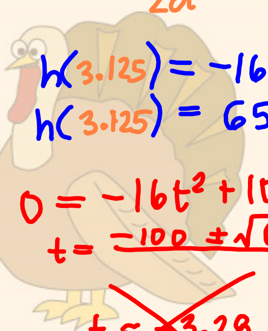
c) The velocity of the object can be modeled by the equation v = -32t + 100, where t is the time in seconds and v is the corresponding velocity of the object. At what velocity does the object hit the ground? v = ? when t ≈ 9.53

a) $h(t) = -16t^2 + 100t + 500$

$t = \frac{-b}{2a} = \frac{-100}{2(-16)} = \frac{-100}{-32} \longrightarrow t = 3.125$

$h(3.125) = -16(3.125)^2 + 100(3.125) + 500$

$h(3.125) = 656.25 \longrightarrow \boxed{\text{max height} = 656.25\text{ ft}}$



b) $0 = -16t^2 + 100t + 500$

$t = \frac{-100 \pm \sqrt{(100)^2 - 4(-16)(500)}}{2(-16)}$

$\cancel{t \approx -3.28}$ or $\boxed{t \approx 9.53\text{ sec}}$

c) $V = -32(9.53) + 100$

$\boxed{V = -204.96\text{ ft/sec}}$

EXAMPLE 6

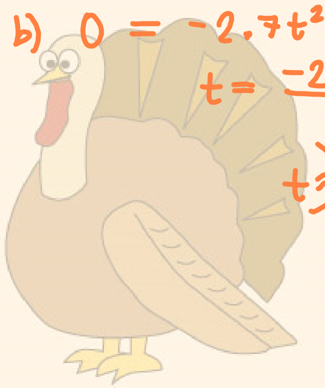
An astronaut standing on the surface of the moon throws a rock into the air with initial velocity of 27 feet per second. The astronaut's hand is 6 feet above the ground when the rock is released. The height of the rock in relation to the time in seconds is given by the equation $h(t) = -2.7t^2 + 27t + 6$.

a) How high did the rock go? y-value of vertex

b) How long is the rock in the air? t = ? when h(t) = 0

a) $t = \frac{-b}{2a} = \frac{-27}{2(-2.7)} = \frac{-27}{-5.4} = 5$

$h(5) = -2.7(5)^2 + 27(5) + 6 \longrightarrow \boxed{\text{max height} = 73.5\text{ ft}}$



b) $0 = -2.7t^2 + 27t + 6$

$t = \frac{-27 \pm \sqrt{(27)^2 - 4(-2.7)(6)}}{2(-2.7)}$

$\cancel{t \approx -.22}$ or $\boxed{t \approx 10.22\text{ sec}}$

Path of a Launched Object Function
$y = -ax^2 + bx + c$

- y is the height of the object
- x is the distance the object traveled

EXAMPLE 7

In firefighting, a good water stream can be modeled by $y = -0.003x^2 + 0.62x + 3$, where x is the horizontal distance traveled in feet and y is the vertical distance traveled in feet.

a) How far can the water travel? find x when y=0

b) What is the maximum height the stream of water will reach? y-value of vertex

a) $0 = -0.003x^2 + 0.62x + 3$

$$x = \frac{-.62 \pm \sqrt{(0.62)^2 - 4(-0.003)(3)}}{2(-0.003)}$$

$x \approx \cancel{-4.73}$ $\boxed{x \approx 211.40 \text{ ft}}$

b) $x = \frac{-b}{2a} = \frac{-.62}{2(-.003)} = \frac{-.62}{-.006} \approx 103.33$

$y = -0.003(103.33)^2 + 0.62(103.33) + 3$

$\boxed{\text{max height} \approx 35.03 \text{ ft}}$

EXAMPLE 8

On September 10, 1960, Mickey Mantle hit the longest home run ever recorded in regular season major league baseball. In a game between the New York Yankees and the Detroit Tigers at Briggs Stadium in Detroit, he sent the ball into a parabolic path that can be modeled by the equation $y = -0.0014x^2 + .9x$, where x is the horizontal distance in feet and y the vertical distance in feet of the ball from home plate.

a) How far did the ball land from home plate?

b) What was the maximum height reached by the ball?