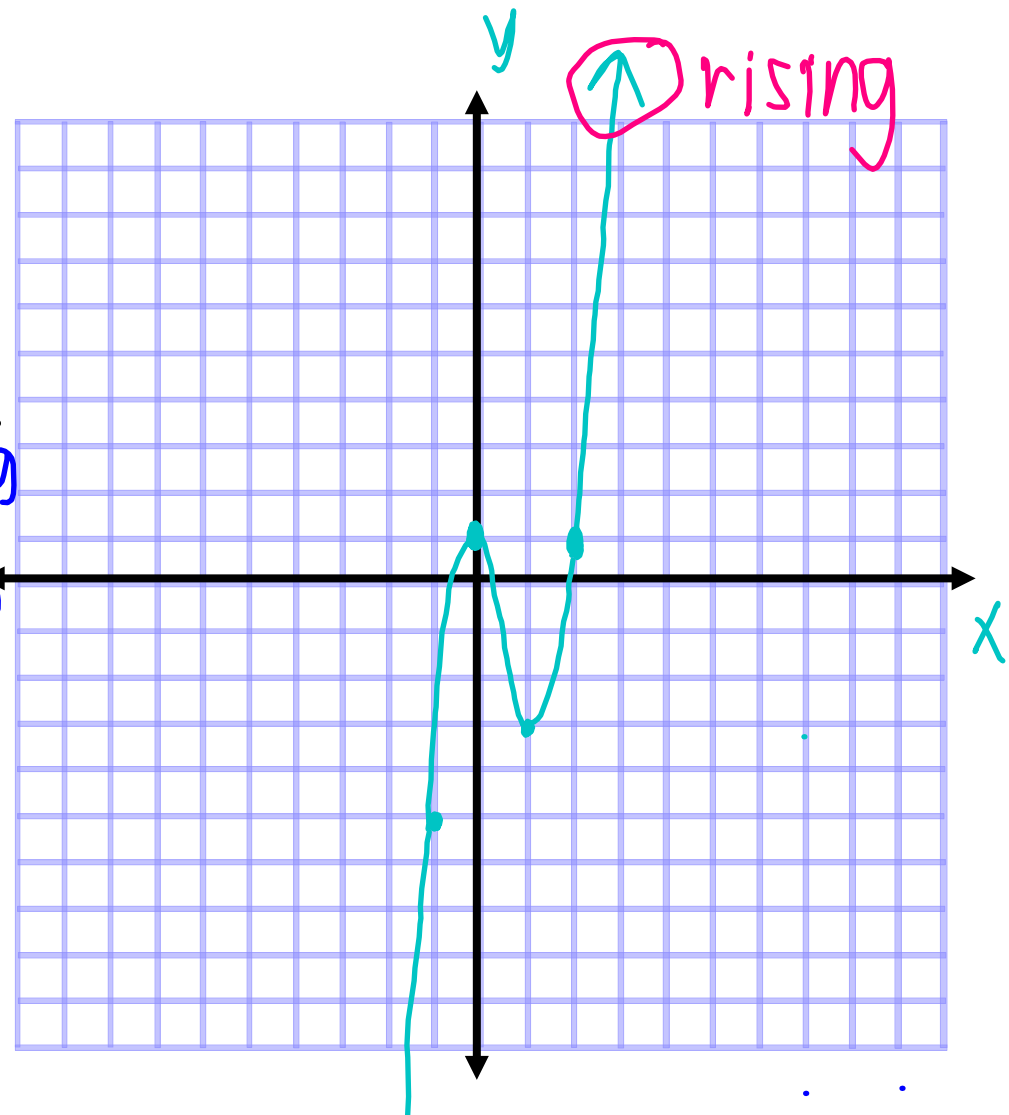
3.2 GRAPHING POLYNOMIALS

There are several ways to graph polynomial functions. We'll start by making a table of values. Let's use x = -3, -2, -1, 0, 1, 2, 3.

Graph the function below.
$f(x) = 3x^3 - 5x^2 - 2x + 1$

x		y
-3	$3(-3)^3 - 5(-3)^2 - 2(-3) + 1$	-119
-2	$3(-2)^3 - 5(-2)^2 - 2(-2) + 1$	-39
-1	$3(-1)^3 - 5(-1)^2 - 2(-1) + 1$	-5
0	$3(0)^3 - 5(0)^2 - 2(0) + 1$	1
1	$3(1)^3 - 5(1)^2 - 2(1) + 1$	-3
2	$3(2)^3 - 5(2)^2 - 2(2) + 1$	1
3	$3(3)^3 - 5(3)^2 - 2(3) + 1$	31



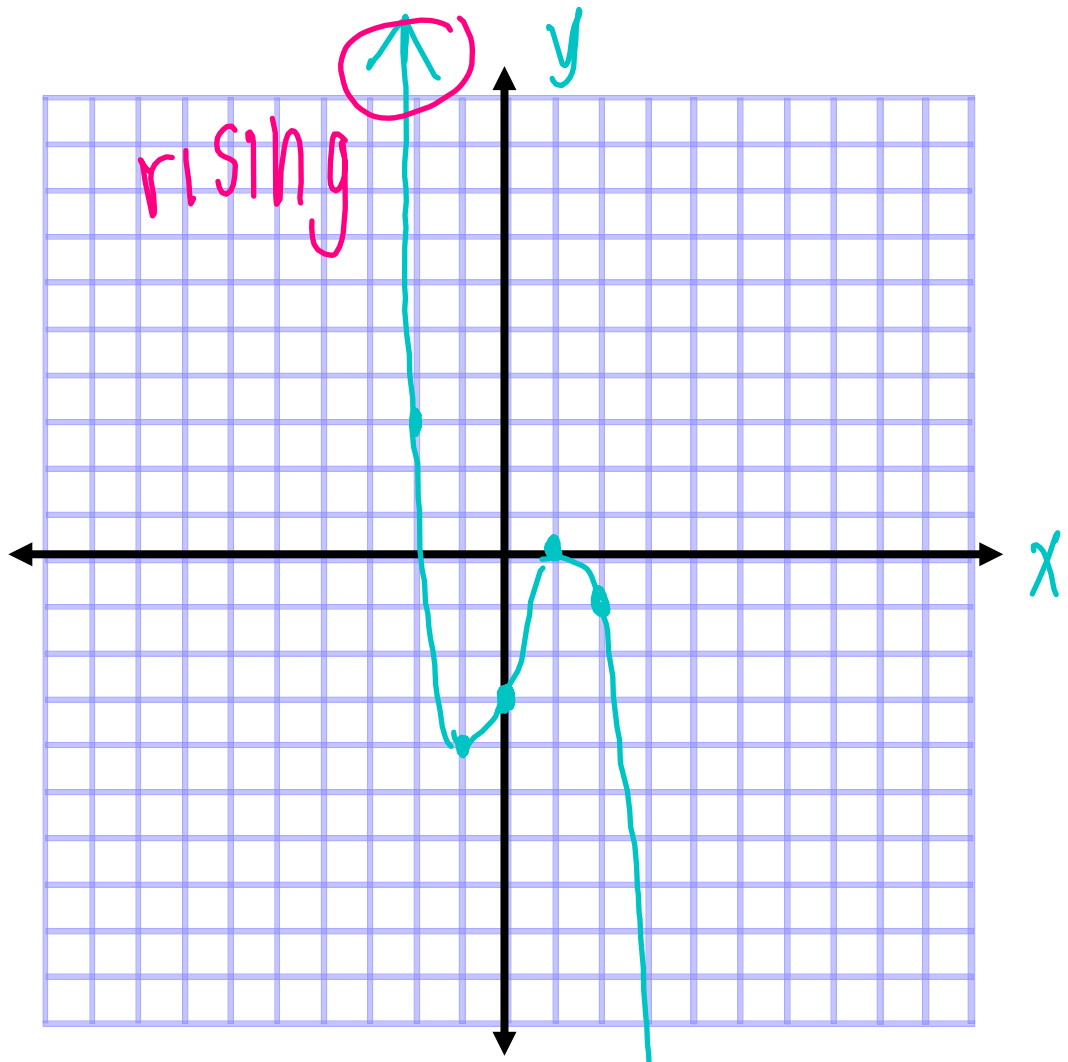
What is the shape of the graph? N-shape
How many U-turns are there? 2
Is the degree even or odd? odd
Is the leading coefficient pos. or neg.? pos.
Describe the end behavior.

$f(x) \to -\infty$ when $x \to -\infty$
$f(x) \to \infty$ when $x \to \infty$

Graph the function below.

$f(x) = -x^3 + x^2 + 3x - 3$

x		y
-3	$-(-3)^3+(-3)^2+3(-3)-3$	24
-2	$-(-2)^3+(-2)^2+3(-2)-3$	3
-1	$-(-1)^3+(-1)^2+3(-1)-3$	-4
0	$-(0)^3+(0)^2+3(0)-3$	-3
1	$-(1)^3+(1)^2+3(1)-3$	0
2	$-(2)^3+(2)^2+3(2)-3$	-1
3	$-(3)^3+(3)^2+3(3)-3$	-12



What is the shape of the graph? backwards N

How many U-turns are there? 2

Is the degree even or odd? odd

Is the leading coefficient pos. or neg.? neg.

Describe the end behavior.

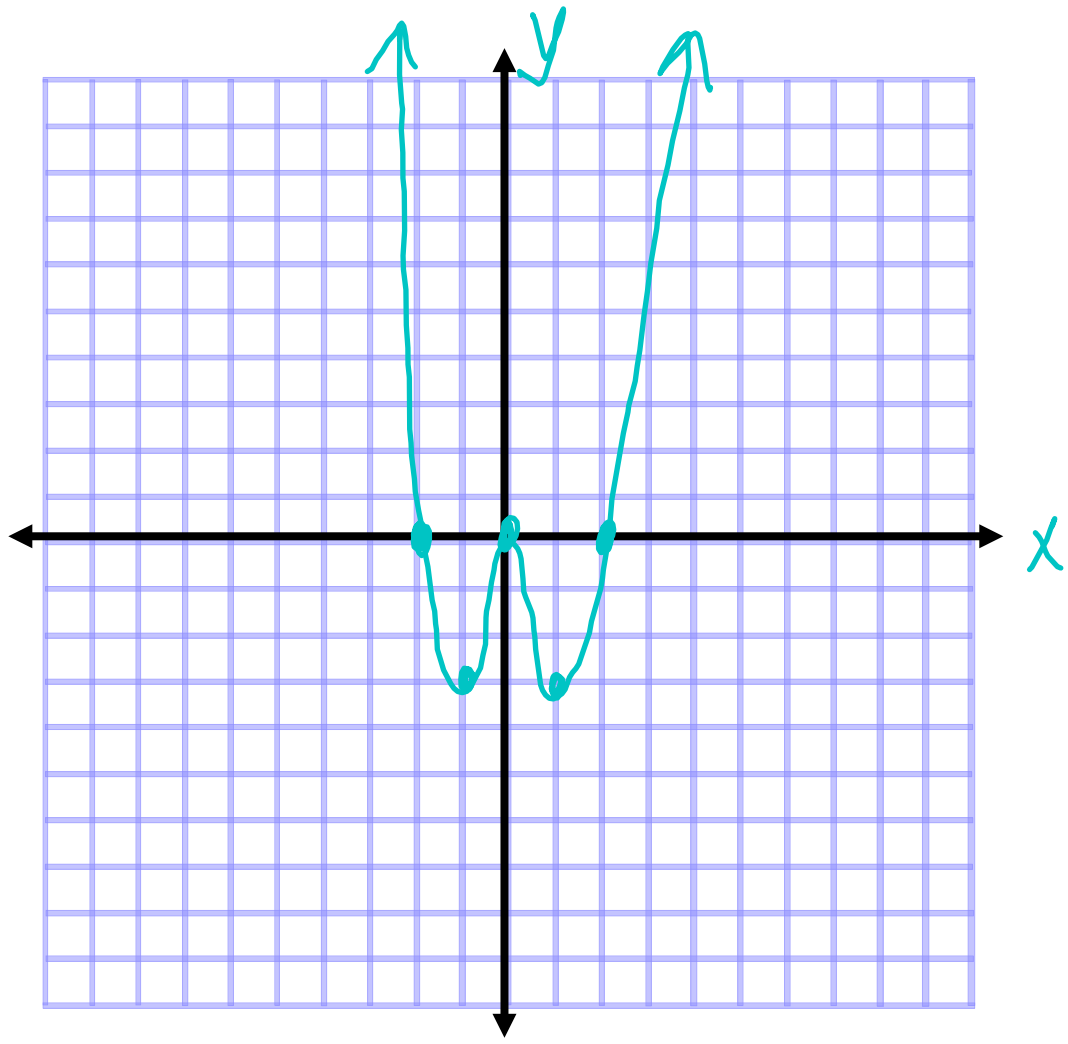
$f(x) \to \infty$ as $x \to -\infty$

$f(x) \to -\infty$ as $x \to \infty$

Graph the function below.

$f(x) = x^4 - 4x^2$

x		y
-3	$(-3)^4 - 4(-3)^2$	45
-2	$(-2)^4 - 4(-2)^2$	0
-1	$(-1)^4 - 4(-1)^2$	-3
0	$(0)^4 - 4(0)^2$	0
1	$(1)^4 - 4(1)^2$	-3
2	$(2)^4 - 4(2)^2$	0
3	$(3)^4 - 4(3)^2$	45



What is the shape of the graph? W - shape

How many U-turns are there? 3

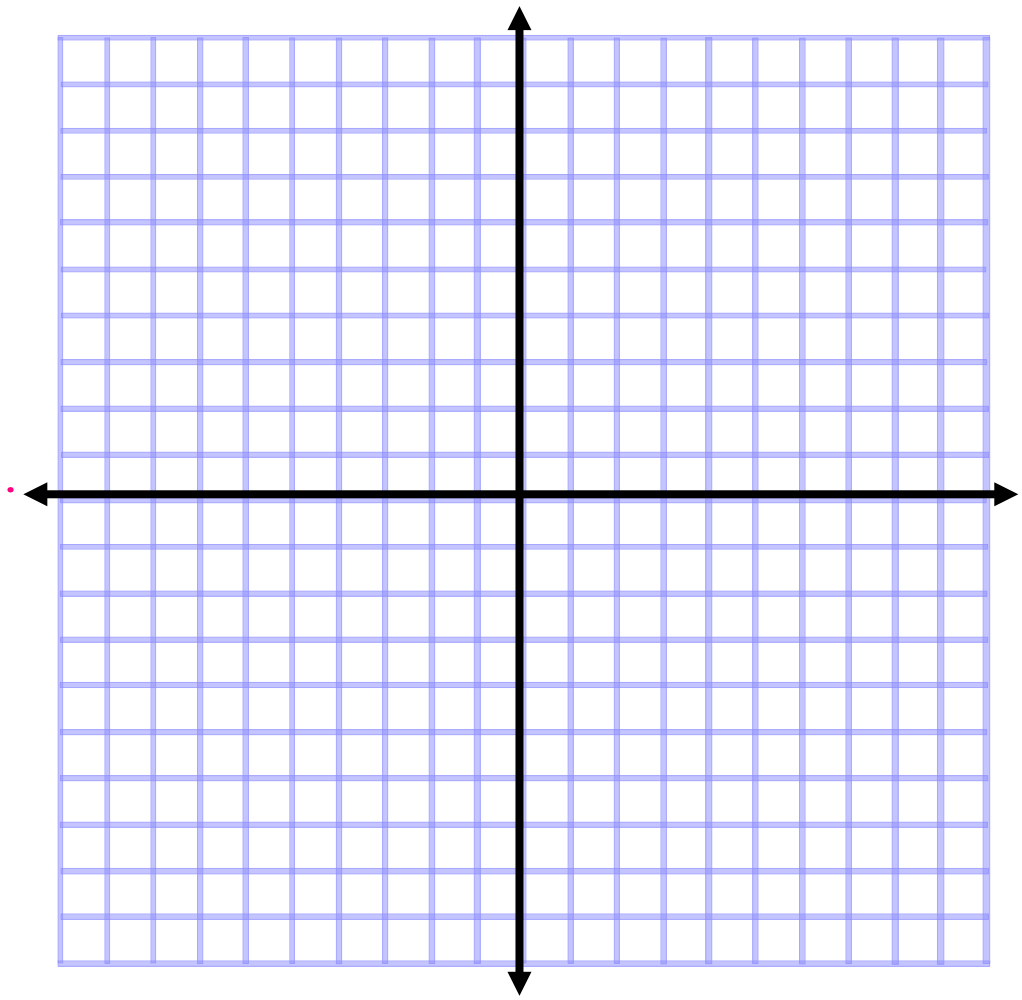
Is the degree even or odd? even

Is the leading coefficient pos. or neg.? pos.

Describe the end behavior. $f(x) \to \infty$ as $x \to \pm\infty$

Graph the function below.

$f(x) = -x^4 + x^3 + 4x^2 - 4$



What is the shape of the graph? M

How many U-turns are there? 3

Is the degree even or odd? even

Is the leading coefficient pos. or neg.? neg

Describe the end behavior. $f(x) \to -\infty$ as $x \to \pm\infty$

	Function	Degree	Name	Sketch of General Shape $a_n > 0$	Sketch of General Shape $a_n < 0$	Max. Turns	Number of Zeros	End Behavior $a_n > 0$	End Behavior $a_n < 0$
★	$y = -5$	0		↔					
★	$y = 3x + 2$	1	linear	↗	↘	0		$f(x) \to \infty$ as $x \to \infty$	$f(x) \to \infty$ as $x \to \infty$
★	$y = x^2 + x - 2$	2	quadratic	∪	∩	1		$f(x) \to \infty$ as $x \to \pm\infty$	
★	$y = 3x^3 - 12x + 4$	3	cubic	↗	↘	2			
★	$y = x^4 + 2x^3 - 5x^2 - 6x$	4	quartic	∪	∩	3			
★	$y = 6x^5 + 5x^4 - 15x^3 - 10x^2 + 5x + 2$	5	quintic	↗	↘	4			

END BEHAVIOR OF A POLYNOMIAL FUNCTION

	***a* is positive**		***a* is negative**	
	left	right	left	right
n is even	rising	rising	falling	falling
n is odd	falling	rising	rising	falling

Practice: Describe the end behavior of each function below.

1. $-2x^5 + 3x^2 - x - 5$

$f(x) \to \infty$ as $x \to -\infty$

$f(x) \to -\infty$ as $x \to \infty$

2. $6x^4 + x^3 - 2x^2 - 4x + 1$

$f(x) \to \infty$ as $x \to \pm\infty$

3. $-7x^6 + 8x^3 - 5$

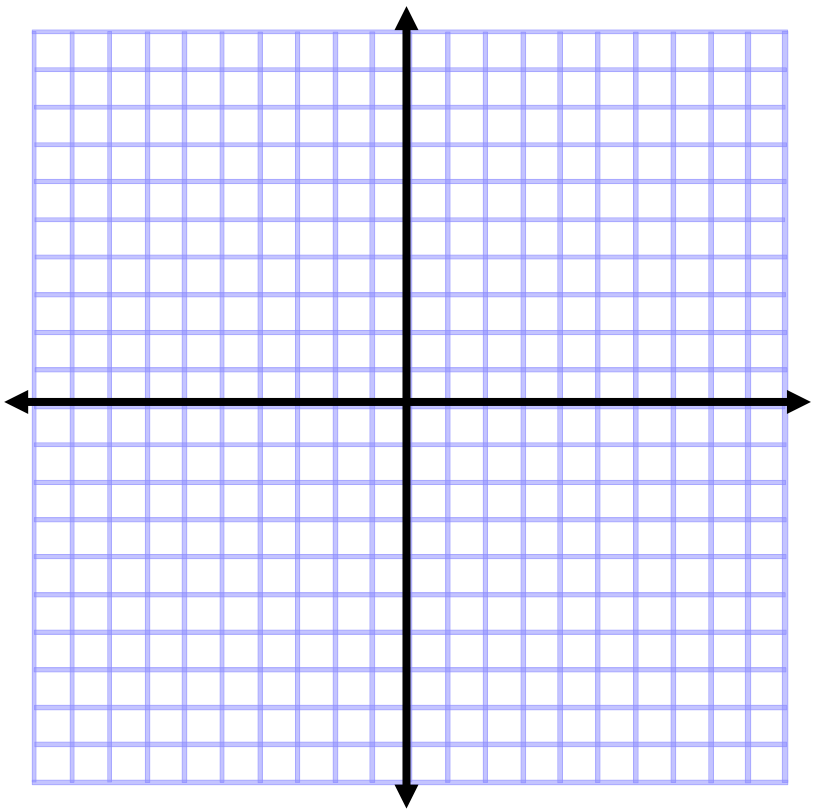
$f(x) \to -\infty$ as $x \to \pm\infty$

4. $5x^3 + x^2 - x - 9$

$f(x) \to -\infty$ as $x \to -\infty$

$f(x) \to \infty$ as $x \to \infty$

What does it mean to be a **zero** of a polynomial?



If *k* is a **zero** of a polynomial, then:

1. x - *k* is a factor of P(x).

2. x = *k* is a solution/root (real or imaginary) of the equation P(x) =0.

3. If *k* is a real root, then *k* is an x-intercept of the graph of P(x).