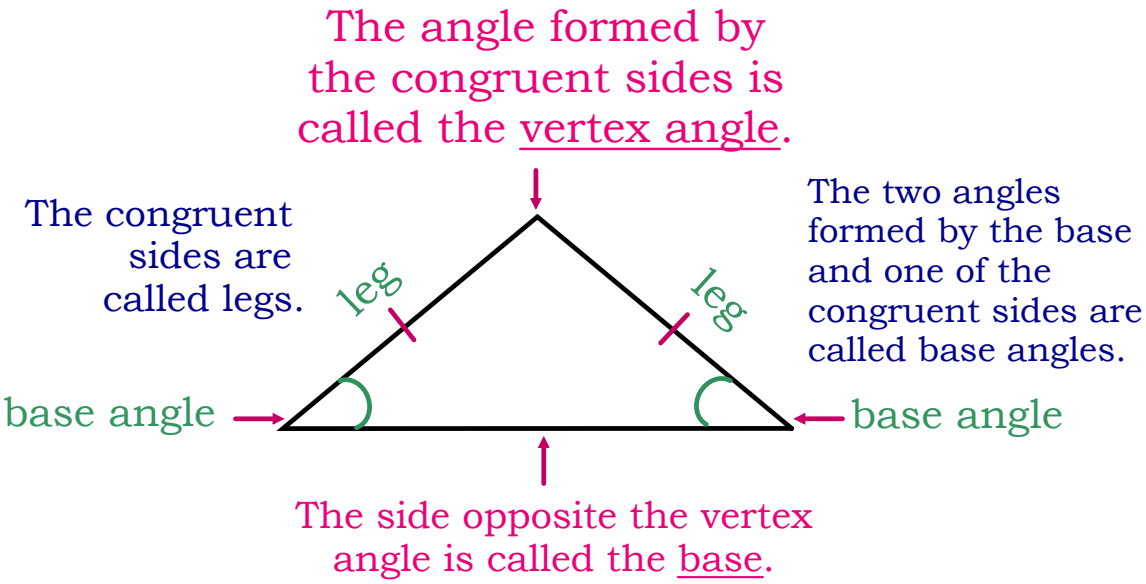
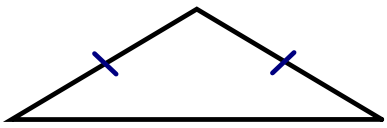
4.6 Isosceles, Equilateral, & Right Triangles

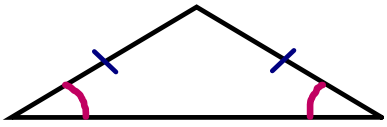
ISOSCELES TRIANGLE



Theorem 4.6: Base Angle Theorem

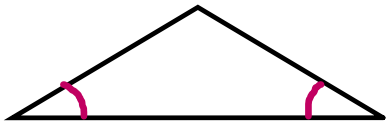
If two sides of a triangle are congruent, then the angles opposite them are congruent.

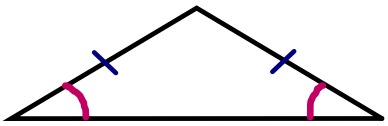
If 

then 

Theorem 4.7: Converse of Base Angle Theorem

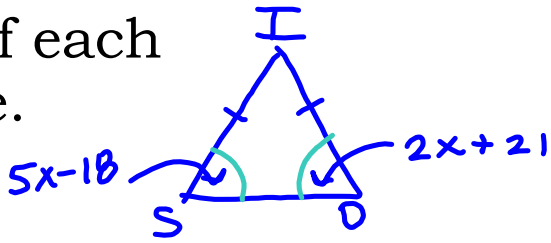
If two angles of a triangle are congruent, then the sides opposite them are congruent.

If 

then 

Example 1

In isosceles △ISO with base SO, m∠S = 5x - 18 and m∠O = 2x + 21. Find the measure of each angle of the triangle.



$5x - 18 = 2x + 21$

$-2x \quad -2x$

$3x - 18 = 21$

$+18 \quad +18$

$\frac{3x}{3} = \frac{39}{3}$

$x = 13$

$m\angle S = 5(13) - 18$

$m\angle S = 47°$

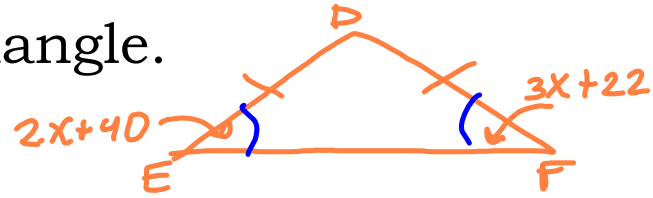
$m\angle O = 2(13) + 21$

$m\angle O = 47°$

$m\angle I = 86°$

Example 2

In isosceles $\triangle DEF$, $\angle D$ is the vertex angle. If $m\angle E = 2x + 40$ and $m\angle F = 3x + 22$, find the measure of each angle of the triangle.



$2x + 40 = 3x + 22$
$-2x \quad -2x$
$40 = x + 22$
$-22 \quad -22$
$18 = x$

$m\angle E = 2(18) + 40$
$m\angle E = 76°$

$m\angle F = 3(18) + 22$
$m\angle F = 76°$

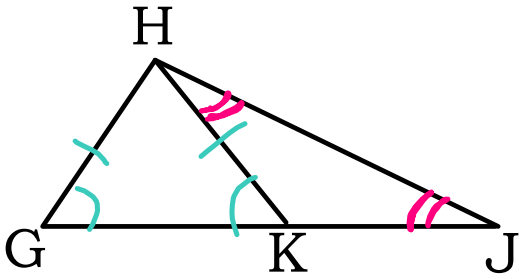
$m\angle D = 28°$

Example 3

Complete the statements below.

a) If $\overline{HG} = \overline{HK}$, then $\angle$ G = $\angle$ GKH.

b) If $\angle KHJ = \angle KJH$, then KH = KJ.



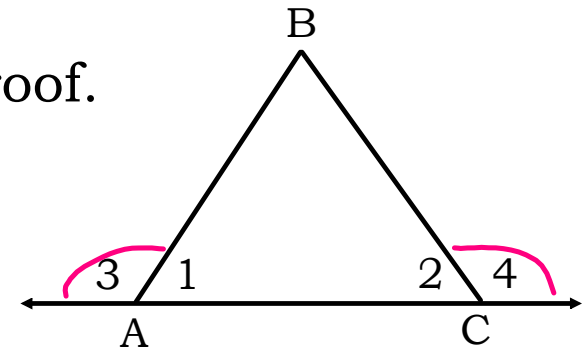
Example 4

Write a two-column proof.

Given: $m\angle 3 = m\angle 4$

Prove: $\overline{AB} \cong \overline{BC}$

Proof:



Statements	Reasons
① $m\angle 3 = m\angle 4$	① given
② $m\angle 1 + m\angle 3 = 180$ $m\angle 2 + m\angle 4 = 180$	② linear pair post.
③ $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 4$	③ subst. prop.
④ $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$	④ subst. prop.
⑤ $m\angle 1 = m\angle 2$	⑤ subtraction prop.
⑥ $\overline{AB} \cong \overline{BC}$	⑥ Base Angle Converse

Corollary to Base Angles Theorem

If a triangle is equilateral,
then it is equiangular.

Corollary to Converse of Base Angles Theorem

If a triangle is equiangular,
then it is equilateral.

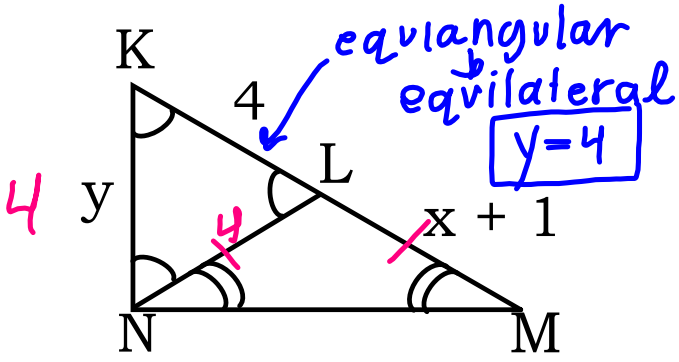
A triangle is equilateral
if and only if
it is equiangular.

REMEMBER:

Each angle of an equilateral triangle measures 60°.

Example 5

Find the values of x and y.



equiangular → equilateral

y = 4

x + 1 = 4

x = 3

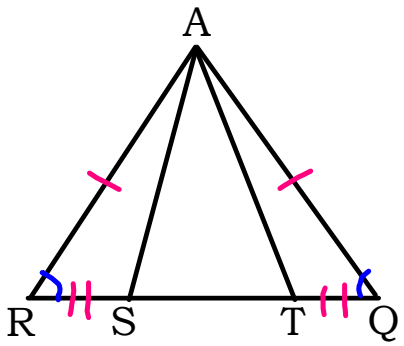
Example 6

Write a two-column proof.

Given: $\overline{AR} \cong \overline{AQ}$, $\overline{RS} \cong \overline{QT}$

Prove: $\overline{AS} \cong \overline{AT}$

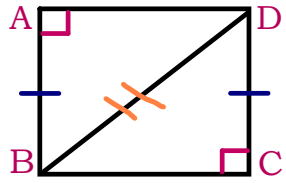
Proof:



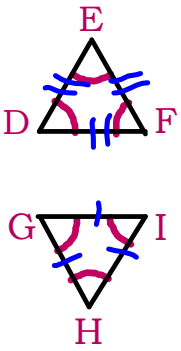
Statements	Reasons
1) $\overline{AR} \cong \overline{AQ}$, $\overline{RS} \cong \overline{QT}$	1) given
2) $\angle R \cong \angle Q$	2) Base Angle Thm.
3) $\triangle ARS \cong \triangle AQT$	3) SAS
4) $\overline{AS} \cong \overline{AT}$	4) CPCTC

Example 7

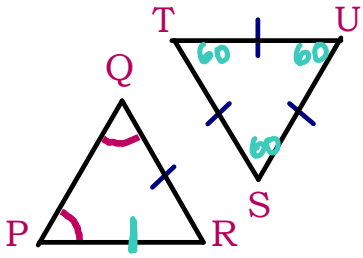
Determine whether there is enough information to prove that the triangles are congruent. Explain your answer.

a) 

$\triangle ABD \cong \triangle CDB$
HL

b) 

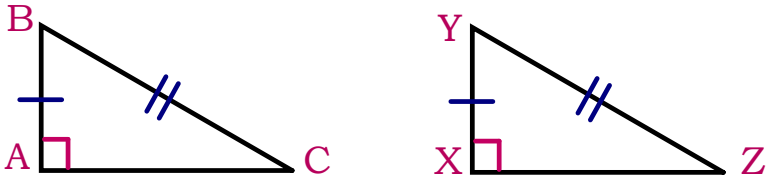
not enough info

c) 

not enough info

Theorem 4.8: Hypotenuse-Leg (HL) Congruence Theorem

If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are congruent.

If 

then $\Delta ABC \cong \Delta XYZ$ by HL