We've been adding, subtracting, multiplying, and factoring polynomials so far. Let's make sure we know what a polynomial is!

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x^1 + a_0$$

This means that a polynomial:

- is typically written in descending order
- has positive integer exponents

Are each of the following a polynomial? If so, state the degree of the polynomial and the leading coefficient.

1. $h(x) = x^4 - \frac{1}{4}x^2 + 3$ polynomial
   degree = 4
   LC = 1
2. $f(x) = 5x^2 + 3x^{-1} - x$
   not a polynomial
3. $k(x) = x + 2^x - 0.6x^5$
   not a polynomial
4. $p(x) = 7x - \sqrt{3} + \pi x^2$ polynomial
   degree = 2
   LC = π

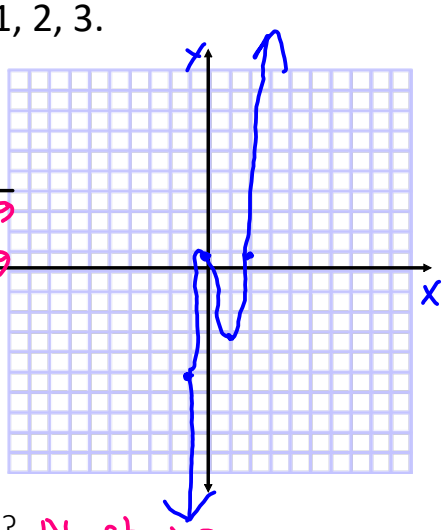
3.2 GRAPHING POLYNOMIALS

There are several ways to graph polynomial functions. We'll start by making a table of values. Let's use x = -3, -2, -1, 0, 1, 2, 3.

Graph the function below.

$f(x) = 3x^3 - 5x^2 - 2x + 1$

x		y
-3	$3(-3)^3-5(-3)^2-2(-3)+1$	-119
-2	$3(-2)^3-5(-2)^2-2(-2)+1$	-39
-1	$3(-1)^3-5(-1)^2-2(-1)+1$	-5
0	$3(0)^3-5(0)^2-2(0)+1$	1
1	$3(1)^3-5(1)^2-2(1)+1$	-3
2	$3(2)^3-5(2)^2-2(2)+1$	1
3	$3(3)^3-5(3)^2-2(3)+1$	31



What is the shape of the graph? N-shape

How many U-turns are there? 2

Is the degree even or odd? odd

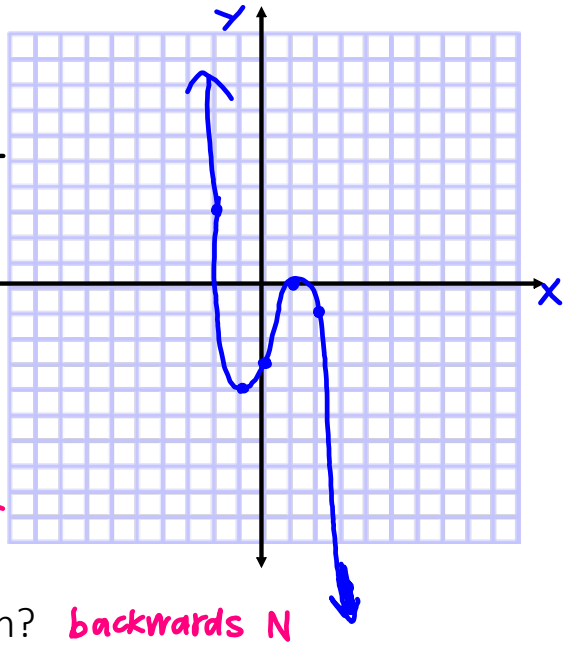
Is the leading coefficient pos. or neg.? positive

Describe the end behavior. $f(x) \to -\infty$ as $x \to -\infty$; $f(x) \to \infty$ as $x \to \infty$

Graph the function below.

$f(x) = -x^3 + x^2 + 3x - 3$

x		y
-3	$-(-3)^3+(-3)^2+3(-3)-3$	24
-2	$-(-2)^3+(-2)^2+3(-2)-3$	3
-1	$-(-1)^3+(-1)^2+3(-1)-3$	-4
0	$-(0)^3+(0)^2+3(0)-3$	-3
1	$-(1)^3+(1)^2+3(1)-3$	0
2	$-(2)^3+(2)^2+3(2)-3$	-1
3	$-(3)^3+(3)^2+3(3)-3$	-12



What is the shape of the graph? backwards N

How many U-turns are there? 2

Is the degree even or odd? odd

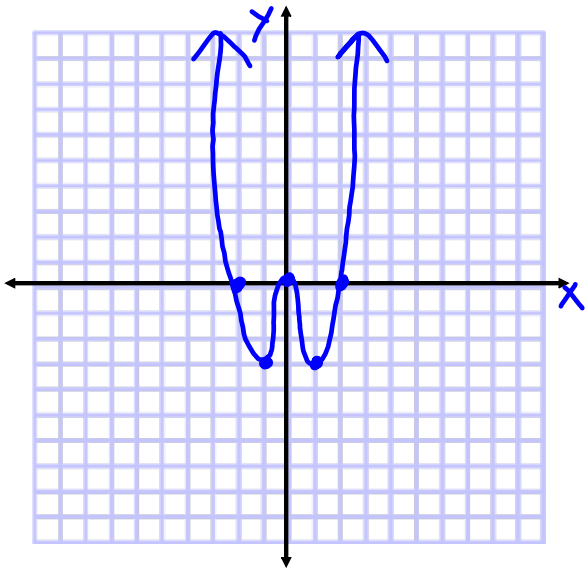
Is the leading coefficient pos. or neg.? negative

Describe the end behavior. $f(x) \to \infty$ as $x \to -\infty$; $f(x) \to -\infty$ as $x \to \infty$

Graph the function below.

$f(x) = x^4 - 4x^2$

x		y
−3	$(-3)^4-4(-3)^2$ $81-36$	45
−2	$(-2)^4-4(-2)^2$ $16-16$	0
−1	$(-1)^4-4(-1)^2$ $1-4$	−3
0	$(0)^4-4(0)^2$	0
1	$(1)^4-4(1)^2$	−3
2	$(2)^4-4(2)^2$	0
3	$(3)^4-4(3)^2$	45



What is the shape of the graph? W-shape

How many U-turns are there? 3

Is the degree even or odd? even

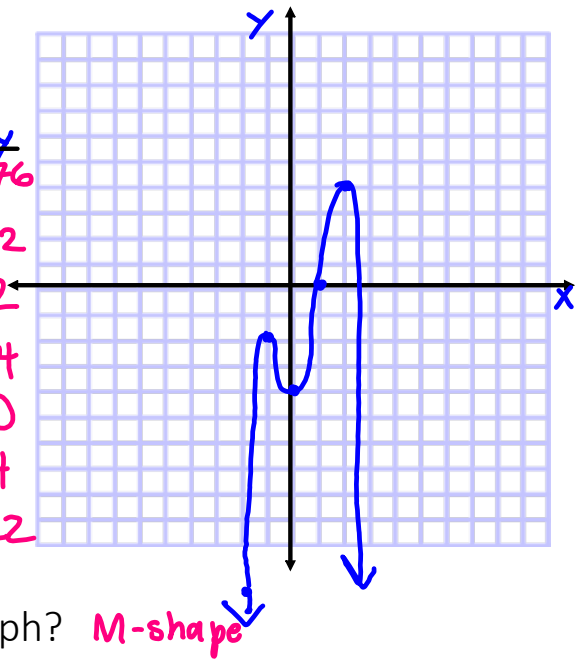
Is the leading coefficient pos. or neg.? positive

Describe the end behavior. $f(x) \to \infty$ as $x \to -\infty$
$f(x) \to \infty$ as $x \to \infty$

Graph the function below.

$f(x) = -x^4 + x^3 + 4x^2 - 4$

x		y
−3	$-(-3)^4+(-3)^3+4(-3)^2-4$ $-81+-27+36-4$	−76
−2	$-(-2)^4+(-2)^3+4(-2)^2-4$	−12
−1	$-(-1)^4+(-1)^3+4(-1)^2-4$	−2
0	$-(0)^4+(0)^3+4(0)^2-4$	−4
1	$-(1)^4+(1)^3+4(1)^2-4$	0
2	$-(2)^4+(2)^3+4(2)^2-4$	4
3	$-(3)^4+(3)^3+4(3)^2-4$ $-81+27+36-4$	−22



What is the shape of the graph? M-shape

How many U-turns are there? 3

Is the degree even or odd? even

Is the leading coefficient pos. or neg.? negative

Describe the end behavior. $f(x) \to -\infty$ as $x \to -\infty$
$f(x) \to -\infty$ as $x \to \infty$

	Function	Degree	Name	Sketch of General Shape $a_n > 0$	Sketch of General Shape $a_n < 0$	Max. Turns	Number of Zeros	End Behavior $a_n > 0$	End Behavior $a_n < 0$
★	$y = -5$	0	constant	↔		0			
★	$y = 3x + 2$	1	linear	↗		0	1	$f(x) \to -\infty$ as $x \to -\infty$; $f(x) \to \infty$ as $x \to \infty$	
★	$y = x^2 + x - 2$	2	quadratic	∪		1	2	$f(x) \to \infty$ as $x \to -\infty$; $f(x) \to \infty$ as $x \to \infty$	
★	$y = 3x^3 - 12x + 4$	3	cubic	↗∿↘		2	3	$f(x) \to -\infty$ as $x \to -\infty$; $f(x) \to \infty$ as $x \to \infty$	
★	$y = x^4 + 2x^3 - 5x^2 - 6x$	4	quartic	W		3	4	$f(x) \to \infty$ as $x \to -\infty$; $f(x) \to \infty$ as $x \to \infty$	
★	$y = 6x^5 + 5x^4 - 15x^3 - 10x^2 + 5x + 2$	5	quintic	↗∿∿↘		4	5	$f(x) \to -\infty$ as $x \to -\infty$; $f(x) \to \infty$ as $x \to \infty$	

END BEHAVIOR OF A POLYNOMIAL FUNCTION

	leading coefficient: a is positive		a is negative	
	left	right	left	right
n is even quadratic	$f(x) \to \infty$ as $x \to -\infty$	$f(x) \to \infty$ as $x \to \infty$	$f(x) \to -\infty$ as $x \to -\infty$	$f(x) \to -\infty$ as $x \to \infty$
degree n is odd linear	$f(x) \to -\infty$ as $x \to -\infty$	$f(x) \to \infty$ as $x \to \infty$	$f(x) \to \infty$ as $x \to -\infty$	$f(x) \to -\infty$ as $x \to \infty$

Practice: Describe the end behavior of each function below.

1. $-2x^5 + 3x^2 - x - 5$ (odd) $f(x) \to \infty$ as $x \to -\infty$; $f(x) \to -\infty$ as $x \to \infty$

2. $6x^4 + x^3 - 2x^2 - 4x + 1$ (even) $f(x) \to \infty$ as $x \to -\infty$; $f(x) \to \infty$ as $x \to \infty$

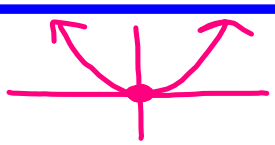
3. $-7x^6 + 8x^3 - 5$ (even) $f(x) \to -\infty$ as $x \to -\infty$; $f(x) \to -\infty$ as $x \to \infty$

4. $5x^3 + x^2 - x - 9$ (odd) $f(x) \to -\infty$ as $x \to -\infty$; $f(x) \to \infty$ as $x \to \infty$

Multiplicity of a Root

The multiplicity of a root is the same as the exponent on the factor.

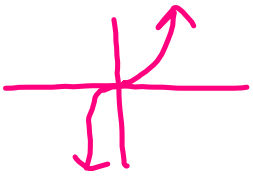
$f(x) = x^2$



What happens to the graph at x = 0?

If the multiplicity is even, it "bounces" off the axis.

$f(x) = x^3$



What happens to the graph at x = 0?

If the multiplicity is odd, it crosses through the axis.

Sketch the graph of the function using its intercepts.

$f(x) = \frac{1}{6}(x + 3)(x - 2)^2$

Degree: 3

End Behavior: $f(x) \to -\infty$ as $x \to -\infty$; $f(x) \to \infty$ as $x \to \infty$

Zeros: $x = -3$ cross; $x = 2$ bounce

Max. # of turns: 2

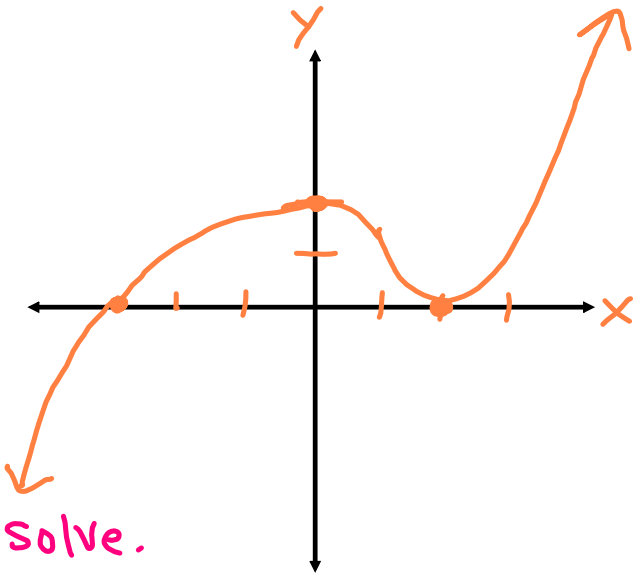
y-intercept: set $x = 0$ & solve.

$y = \frac{1}{6}(0+3)(0-2)^2$

$y = \frac{1}{6}(3)(-2)^2$

$y = \frac{1}{6}(3)(4)$

$y = 2$



Sketch the graph of the function using its intercepts.

$f(x) = -(x + 3)^2(x - 1)(x - 4)$

Degree: 4

End Behavior: $f(x) \to -\infty$ as $x \to -\infty$; $f(x) \to -\infty$ as $x \to \infty$

Zeros: $x = -3$ bounce; $x = 1$ cross; $x = 4$ cross

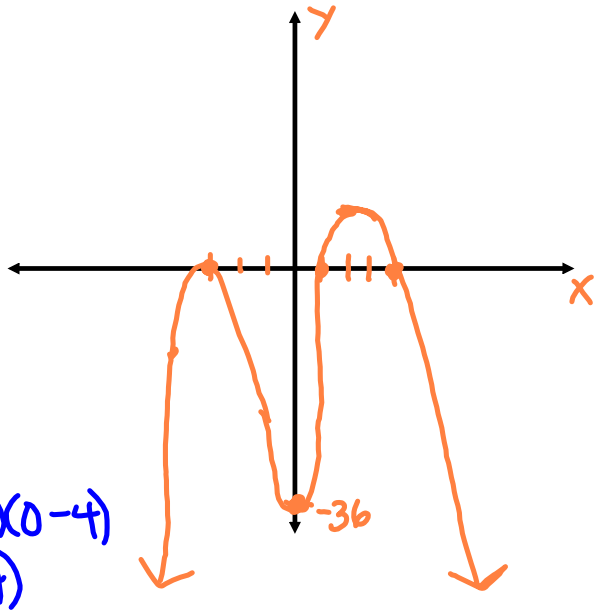
Max. # of turns: 3

y-intercept: $y = -(0+3)^2(0-1)(0-4)$

$y = -(3)^2(-1)(-4)$

$y = -(9)(-1)(-4)$

$y = -36$

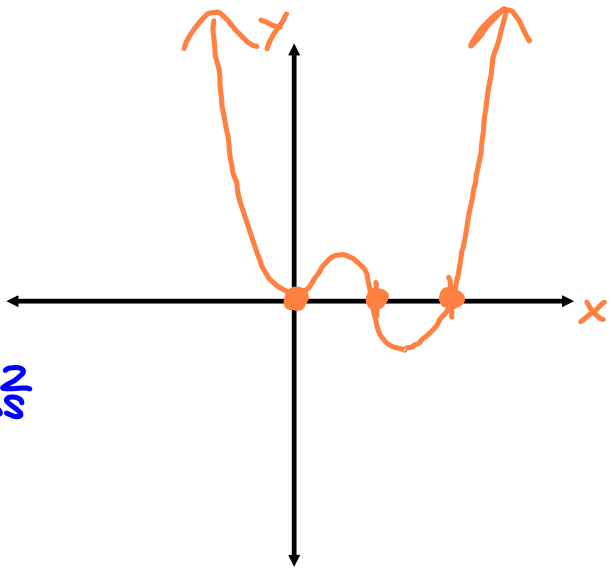


Sketch the graph of the function using its intercepts.

$f(x) = x^2(x^2 - 3x + 2)$

$f(x) = x^2(x-1)(x-2)$

$f(x) = x^4 - 3x^3 + 2x^2$



Degree: 4

End Behavior: $f(x) \to \infty$ as $x \to -\infty$; $f(x) \to \infty$ as $x \to \infty$

Zeros: $x = 0$ bounce; $x = 1$ cross; $x = 2$ cross

Max. # of turns: 3

y-intercept: $y = 0$

Sketch the graph of the function using its intercepts.

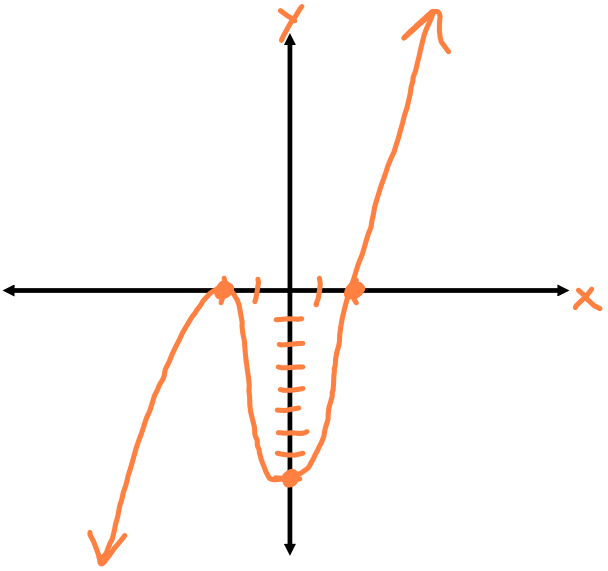
$f(x) = x^2(x+2) - 4(x+2)$

$f(x) = (x+2)(x^2 - 4)$

$f(x) = (x^3 + 2x^2)(-4x - 8)$

$f(x) = (x+2)(x+2)(x-2)$

$f(x) = (x+2)^2(x-2)$



Degree: 3

End Behavior: $f(x) \to -\infty$ as $x \to -\infty$; $f(x) \to \infty$ as $x \to \infty$

Zeros: $x = -2$ bounce; $x = 2$ cross

Max. # of turns: 2

y-intercept: $y = -8$