

We've been adding, subtracting, multiplying, and factoring polynomials so far. Let's make sure we know what a polynomial is!

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x^1 + a_0$$

This means that a polynomial:

- is typically written in **descending order**
- has **positive integer exponents**

Are each of the following a polynomial? If so, state the degree of the polynomial and the leading coefficient.

1. $h(x) = x^4 - \frac{1}{4}x^2 + 3$ **polynomial**
degree = 4

LC = 1

2. $f(x) = 5x^2 + 3x^{-1} - x$
not a polynomial

3. $k(x) = x + 2^x + 0.6x^5$
not a polynomial

4. $p(x) = 7x - \sqrt{3} + \pi x^2$ **polynomial**
degree = 2
LC = π

3.2 GRAPHING POLYNOMIALS

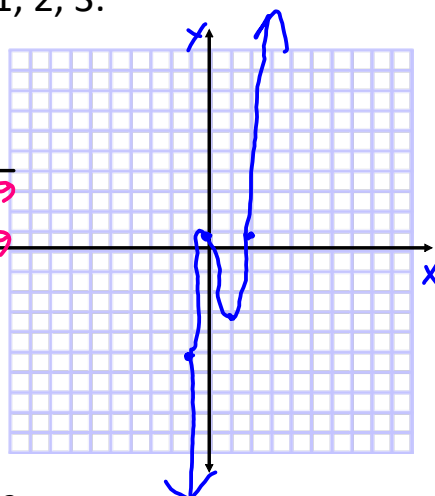
There are several ways to graph polynomial functions. We'll start by making a **table of values**.

Let's use $x = -3, -2, -1, 0, 1, 2, 3$.

Graph the function below.

$$f(x) = 3x^3 - 5x^2 - 2x + 1$$

x	y
-3	$3(-3)^3 - 5(-3)^2 - 2(-3) + 1 = -119$
-2	$3(-2)^3 - 5(-2)^2 - 2(-2) + 1 = -39$
-1	$3(-1)^3 - 5(-1)^2 - 2(-1) + 1 = -5$
0	$3(0)^3 - 5(0)^2 - 2(0) + 1 = 1$
1	$3(1)^3 - 5(1)^2 - 2(1) + 1 = -3$
2	$3(2)^3 - 5(2)^2 - 2(2) + 1 = 1$
3	$3(3)^3 - 5(3)^2 - 2(3) + 1 = 31$



What is the shape of the graph?

N-shape

How many U-turns are there?

2

Is the degree even or odd?

odd

Is the leading coefficient pos. or neg.?

positive

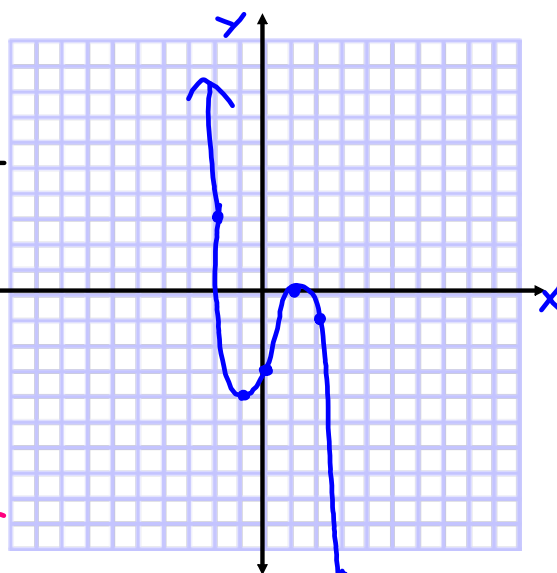
Describe the end behavior.

*$f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$
 $f(x) \rightarrow \infty$ as $x \rightarrow \infty$*

Graph the function below.

$$f(x) = -x^3 + x^2 + 3x - 3$$

x	y
-3	$-(-3)^3 + (-3)^2 + 3(-3) - 3 = 24$
-2	$-(-2)^3 + (-2)^2 + 3(-2) - 3 = 3$
-1	$-(-1)^3 + (-1)^2 + 3(-1) - 3 = -4$
0	$-(0)^3 + (0)^2 + 3(0) - 3 = -3$
1	$-(1)^3 + (1)^2 + 3(1) - 3 = 0$
2	$-(2)^3 + (2)^2 + 3(2) - 3 = -1$
3	$-(3)^3 + (3)^2 + 3(3) - 3 = -12$



What is the shape of the graph?

backwards N

How many U-turns are there?

2

Is the degree even or odd?

odd

Is the leading coefficient pos. or neg.?

negative

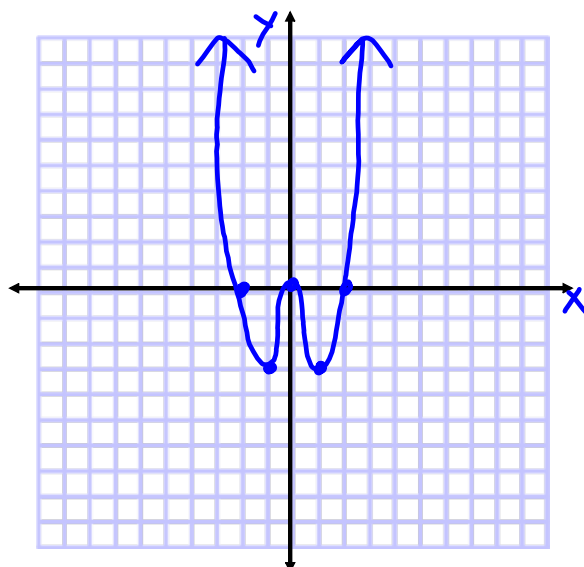
Describe the end behavior.

*$f(x) \rightarrow \infty$ as $x \rightarrow -\infty$
 $f(x) \rightarrow -\infty$ as $x \rightarrow \infty$*

Graph the function below.

$$f(x) = x^4 - 4x^2$$

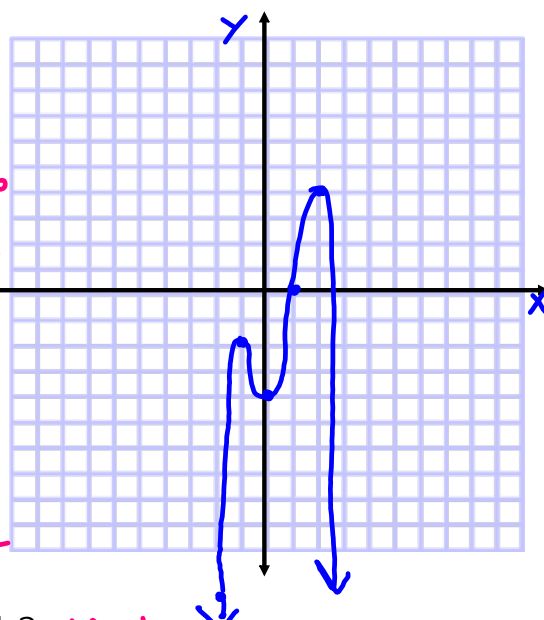
x	y
-3	$(-3)^4 - 4(-3)^2 = 45$
-2	$(-2)^4 - 4(-2)^2 = 0$
-1	$(-1)^4 - 4(-1)^2 = -3$
0	$(0)^4 - 4(0)^2 = 0$
1	$(1)^4 - 4(1)^2 = -3$
2	$(2)^4 - 4(2)^2 = 0$
3	$(3)^4 - 4(3)^2 = 45$

What is the shape of the graph? **W-shape**How many U-turns are there? **3**Is the degree even or odd? **even**Is the leading coefficient pos. or neg.? **positive**Describe the end behavior. $f(x) \rightarrow \infty$ as $x \rightarrow -\infty$
 $f(x) \rightarrow \infty$ as $x \rightarrow \infty$

Graph the function below.

$$f(x) = -x^4 + x^3 + 4x^2 - 4$$

x	y
-3	$-(-3)^4 + (-3)^3 + 4(-3)^2 - 4 = -76$
-2	$-(-2)^4 + (-2)^3 + 4(-2)^2 - 4 = -12$
-1	$-(-1)^4 + (-1)^3 + 4(-1)^2 - 4 = -2$
0	$-(-0)^4 + (0)^3 + 4(0)^2 - 4 = -4$
1	$-(1)^4 + (1)^3 + 4(1)^2 - 4 = 0$
2	$-(2)^4 + (2)^3 + 4(2)^2 - 4 = 4$
3	$-(3)^4 + (3)^3 + 4(3)^2 - 4 = -22$

What is the shape of the graph? **M-shape**How many U-turns are there? **3**Is the degree even or odd? **even**Is the leading coefficient pos. or neg.? **negative**Describe the end behavior. $f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$
 $f(x) \rightarrow -\infty$ as $x \rightarrow \infty$

Function	Degree	Name	Sketch of General Shape		Max. Turns	Number of Zeros	End Behavior	
			$a_n > 0$	$a_n < 0$			$a_n > 0$	$a_n < 0$
★ $y = -5$	0	constant			0			
★ $y = 3x + 2$	1	linear			0	1	$f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$	$f(x) \rightarrow \infty$ as $x \rightarrow \infty$
★ $y = x^2 + x - 2$	2	quadratic			1	2	$f(x) \rightarrow \infty$ as $x \rightarrow -\infty$	$f(x) \rightarrow \infty$ as $x \rightarrow \infty$
★ $y = 3x^3 - 12x + 4$	3	cubic			2	3	$f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$	$f(x) \rightarrow \infty$ as $x \rightarrow \infty$
★ $y = x^4 + 2x^3 - 5x^2 - 6x$	4	quartic			3	4	$f(x) \rightarrow \infty$ as $x \rightarrow -\infty$	$f(x) \rightarrow \infty$ as $x \rightarrow \infty$
★ $y = 6x^5 + 5x^4 - 15x^3 - 10x^2 + 5x + 2$	5	quintic			4	5	$f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$	$f(x) \rightarrow \infty$ as $x \rightarrow \infty$

END BEHAVIOR OF A POLYNOMIAL FUNCTION

	leading coefficient a is positive		a is negative	
	left	right	left	right
n is even quadratic	$f(x) \rightarrow \infty$ as $x \rightarrow -\infty$	$f(x) \rightarrow \infty$ as $x \rightarrow \infty$	$f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$	$f(x) \rightarrow -\infty$ as $x \rightarrow \infty$
degree n is odd linear	$f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$	$f(x) \rightarrow \infty$ as $x \rightarrow \infty$	$f(x) \rightarrow \infty$ as $x \rightarrow -\infty$	$f(x) \rightarrow -\infty$ as $x \rightarrow \infty$