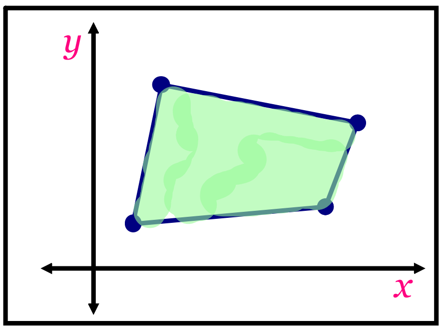
3.4 LINEAR PROGRAMMING & OPTIMIZATION

- Used to find optimal solutions
- Include the following characteristics:

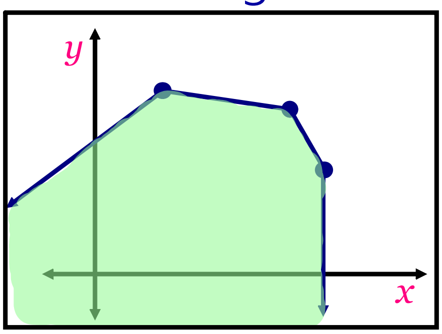
- The inequalities contained in the problem are called constraints.
- The solution to the set of constraints is called the feasible region.
- The function to be maximized or minimized is called the objective function.

CORNER-POINT PRINCIPLE

In linear programming, the maximum and minimum values of the objective function each occur at one of the vertices of the feasible region.



Bounded Region



Unbounded Region

EXAMPLE 1

A. Graph the feasible region.
B. Find the maximum and minimum values, if they exist.

min = 3
max = 12

$P = 3x + y$ ← objective function

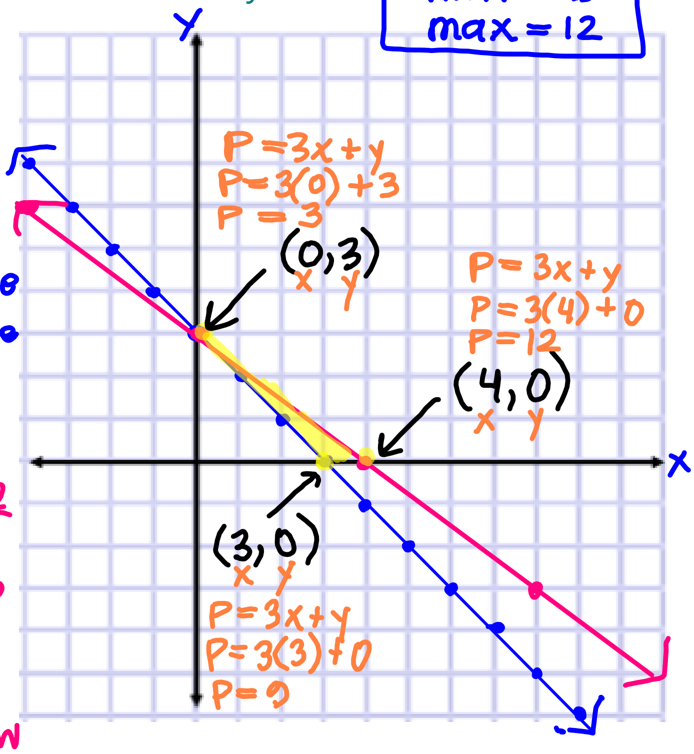
Constraints:

$$\begin{cases} x + y \geq 3 \\ 3x + 4y \leq 12 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

$x \geq 0$: x is positive
$y \geq 0$: y is positive
→ QI

$x + y \geq 3$
$-x \quad -x$
$y \geq -x + 3$
$m = -1$
y-int = 3
shade above

$3x + 4y \leq 12$
$-3x \quad -3x$
$\frac{4y}{4} \leq \frac{-3x}{4} + \frac{12}{4}$
$y \leq -\frac{3}{4}x + 3$
$m = -\frac{3}{4}$
y-int = 3
shade below



EXAMPLE 2

A. Graph the feasible region.
B. Find the maximum and minimum values, if they exist.

unbounded

$E = x + y$

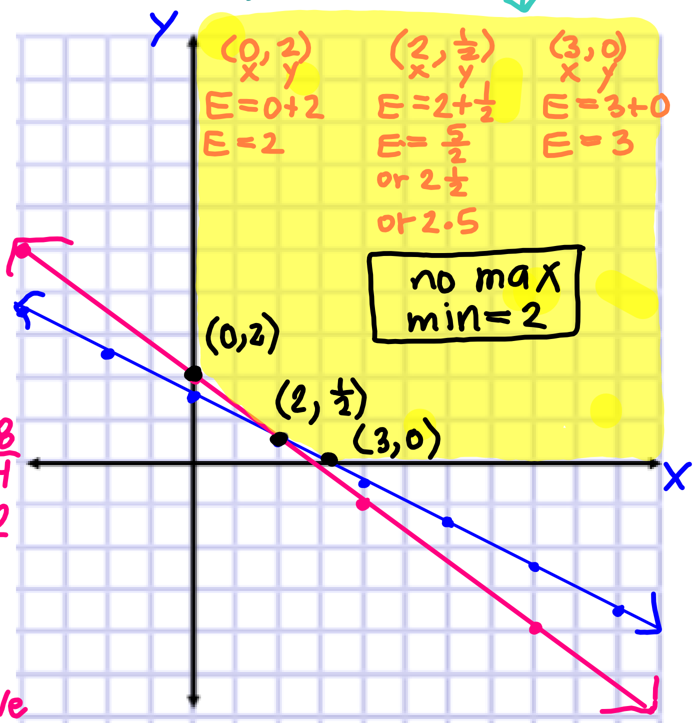
Constraints:

$$\begin{cases} x + 2y \geq 3 \\ 3x + 4y \geq 8 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

QI

$x + 2y \geq 3$
$-x \quad -x$
$\frac{2y}{2} \geq \frac{-x}{2} + \frac{3}{2}$
$y \geq -\frac{1}{2}x + 1.5$
$m = -\frac{1}{2}$
y-int = 1.5
shade above

$3x + 4y \geq 8$
$-3x \quad -3x$
$\frac{4y}{4} \geq \frac{-3x}{4} + \frac{8}{4}$
$y \geq -\frac{3}{4}x + 2$
$m = -\frac{3}{4}$
y-int = 2
shade above



EXAMPLE 3

A. Graph the feasible region.

B. Find the maximum and minimum values, if they exist.

$M = 3x + 2y$

Constraints:

$$\begin{cases} x + y \leq 5 \\ y - x \geq 5 \\ 4x + y \geq -10 \end{cases}$$

$x + y \leq 5$
$-x \quad -x$
$y \leq -x + 5$
below

$y - x \geq 5$
$+x \quad +x$
$y \geq x + 5$
above

$4x + y \geq -10$
$-4x \quad -4x$
$y \geq -4x - 10$
above