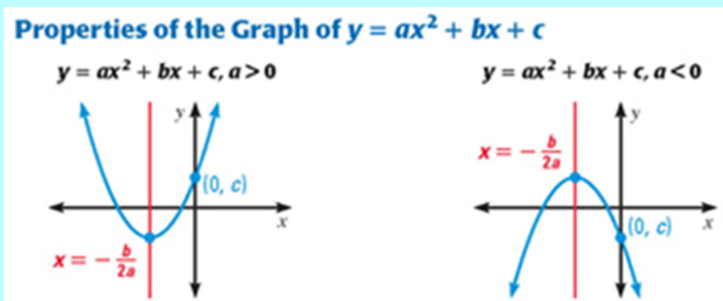


3.1 Graphing Quadratic Functions and Models

Standard (Quadratic) Form: $y = ax^2 + bx + c$

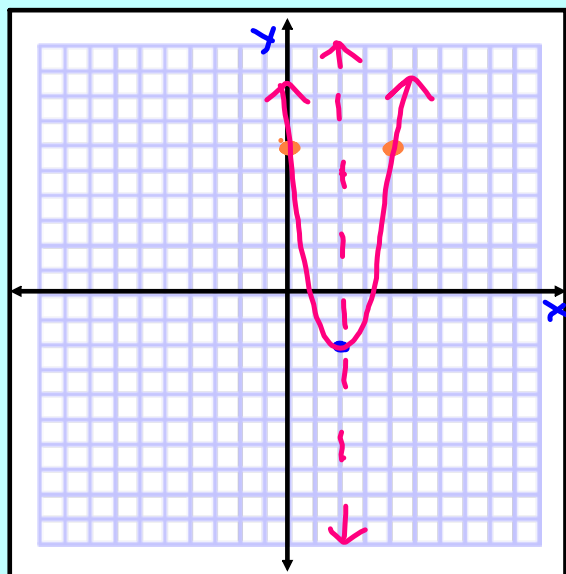
A. Identify characteristics



- The graph opens up if $a > 0$ and down if $a < 0$.
- The graph is narrower than the graph of $y = x^2$ if $a > 1$ and wider if $a < 1$.
- The axis of symmetry is $x = -\frac{b}{2a}$ and the vertex has x -coordinate $-\frac{b}{2a}$.
- The y -intercept is c . So the point $(0, c)$ is on the parabola.

B. Graph Standard Form

Example 1: $f(x) = 2x^2 - 8x + 6$ opens up b/c $a > 0$



$$f(2) = 2(2)^2 - 8(2) + 6 = -2$$

Vertex $(2, -2)$

1. Determine if the graph opens up or down.
2. Find the axis of symmetry.
3. Find the vertex & plot.
4. Find the y -intercept & plot.
5. Reflect the y -intercept over the axis of symmetry.
6. Connect the points with a smooth curve.

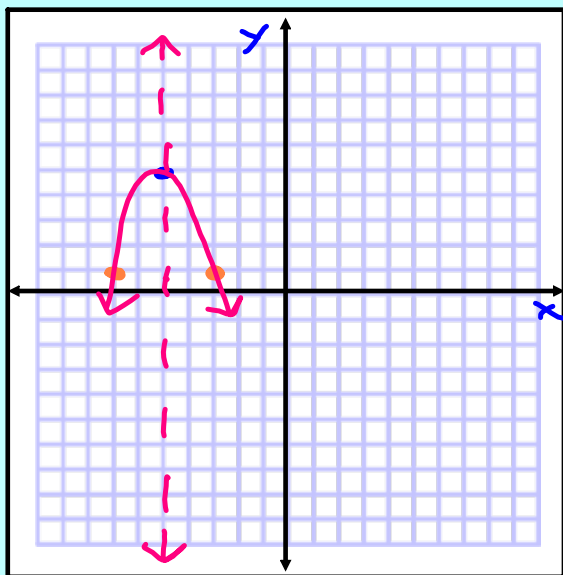
$$x = \frac{-b}{2a} = \frac{8}{2(2)} = \frac{8}{4} = 2$$

a.o.s. $x = 2$

y -int = 6

B. Graph Standard Form

Example 2: $f(x) = -x^2 - 10x - 20$



opens \downarrow $a < 0$

$$x = \frac{-b}{2a} = \frac{10}{2(-1)} = \frac{10}{-2} = -5$$

a.o.s. $x = -5$

$$f(-5) = -(-5)^2 - 10(-5) - 20$$

$$f(-5) = 5 \rightarrow \text{vertex } (-5, 5)$$

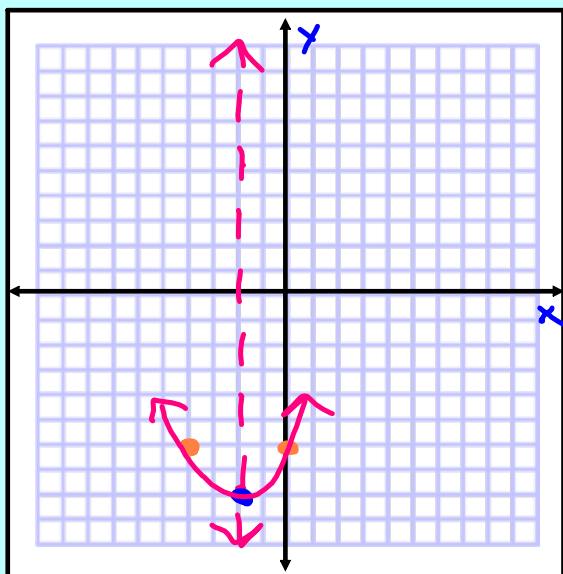
$$y\text{-int} = -20$$

$$f(-3) = -(-3)^2 - 10(-3) - 20$$

$$f(-3) = 1$$

B. Graph Standard Form

Example 3: $f(x) = \frac{1}{2}x^2 + 2x - 6$



opens \uparrow $a > 0$

$$x = \frac{-b}{2a} = \frac{-2}{2(\frac{1}{2})} = \frac{-2}{1} = -2$$

a.o.s. $x = -2$


$$f(-2) = \frac{1}{2}(-2)^2 + 2(-2) - 6$$

$$f(-2) = -8 \text{ vertex } (-2, -8)$$

$$y\text{-int} = -6$$

C. Find the minimum or maximum value

y-value
of
vertex

Example 4: Tell whether the function
opens up

 $y = 3x^2 - 18x + 20$ has a minimum value or a maximum value. Then find the minimum or maximum value.

$$x = \frac{-b}{2a} = \frac{18}{2(3)} = \frac{18}{6} = 3$$

$$y = 3(3)^2 - 18(3) + 20$$

$$y = -7$$

$$\text{min} = -7 \text{ when } x = 3$$

Example 5: Follow the same directions with



$$y = -2x^2 + 4x + 3.$$

$$x = \frac{-b}{2a} = \frac{-4}{2(-2)} = \frac{-4}{-4} = 1$$

$$y = -2(1)^2 + 4(1) + 3$$

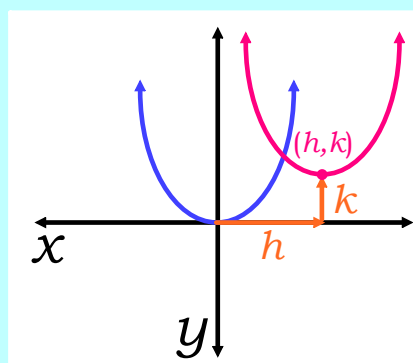
$$y = 5$$

$$\text{max} = 5 \text{ when } x = 1$$

$$\text{Vertex Form: } y = a(x - h)^2 + k$$

↓
 Your book calls
 this standard form.

A. Identify characteristics

**Characteristics:**

- The vertex is (h, k) .
- The axis of symmetry is $x = h$.
- The graph opens up if $a > 0$ and down if $a < 0$.

B. Graph Vertex Form

Example 6Graph $y = -\frac{1}{4}(x + 2)^2 + 5$.

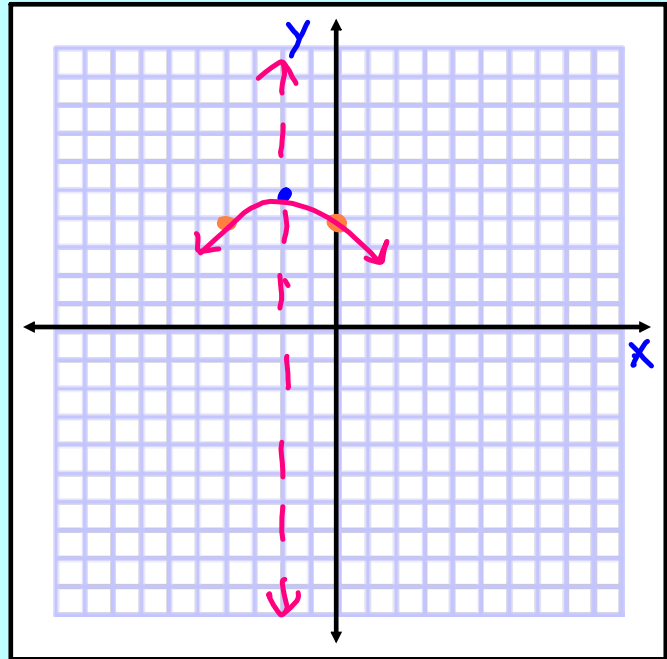
Label the vertex & axis of symmetry.

$$h = -2 \quad k = 5$$

Vertex $(-2, 5)$ a.o.s. $x = -2$

$$y = -\frac{1}{4}(0 + 2)^2 + 5$$

$$y = 4 \quad (0, 4)$$



B. Graph Vertex Form

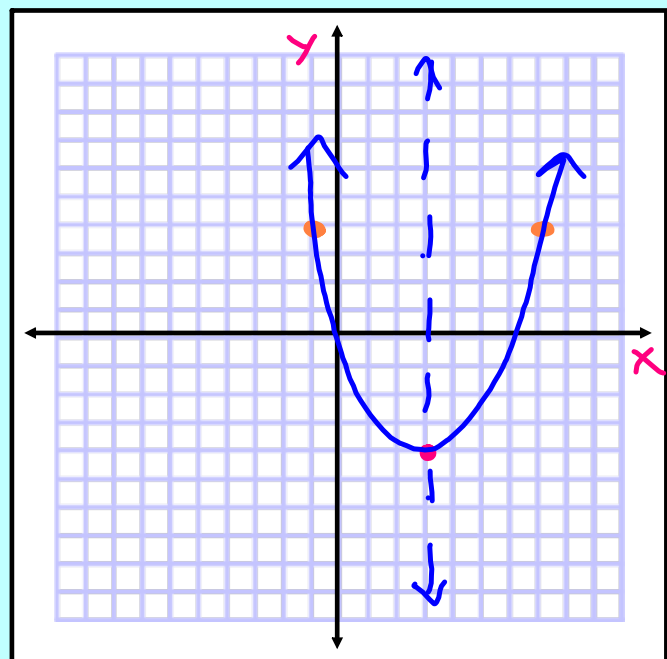
Example 7Graph $y = \frac{1}{2}(x - 3)^2 - 4$.

Label the vertex & axis of symmetry.

vertex $(3, -4)$ a.o.s. $x = 3$

$$y = \frac{1}{2}(7 - 3)^2 - 4$$

$$y = 4 \quad (7, 4)$$



C. Rewrite in Vertex Form

Example 8Given $g(x) = x^2 - 10x + 22$,

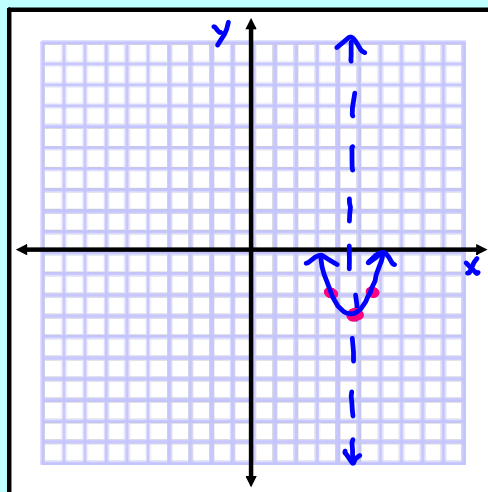
- Write the function in vertex form.
- Give the coordinates of the vertex.
- Write the equation of the axis of symmetry.
- Graph.

a)
$$\begin{array}{r} y = x^2 - 10x + 22 \\ \underline{-22} \qquad \qquad \underline{-22} \\ y - 22 + 25 = x^2 - 10x + 25 \\ \qquad \qquad \qquad \frac{1}{2}(-10) = -5 \\ \qquad \qquad \qquad (-5)^2 = 25 \\ \hline y + 3 = (x - 5)^2 \\ \underline{-3} \qquad \qquad \underline{-3} \\ y = (x - 5)^2 - 3 \end{array}$$

b) (5, -3)
Vertex

c) a.o.s.
 $x = 5$

d) $y = (4 - 5)^2 - 3$
 $y = -2$
(4, -2)



C. Rewrite in Vertex Form

Example 9Given $g(x) = -x^2 + 12x - 37$,

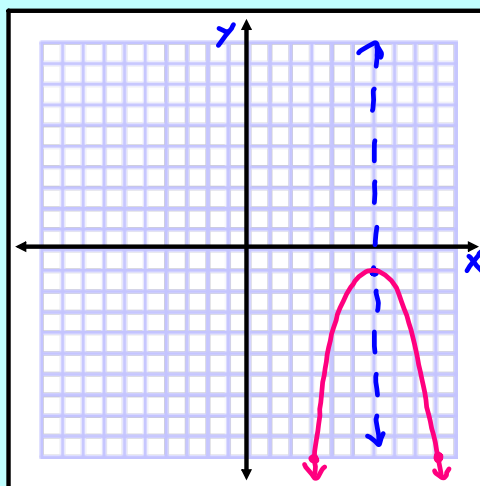
- Write the function in vertex form.
- Give the coordinates of the vertex.
- Write the equation of the axis of symmetry.
- Graph.

a)
$$\begin{array}{r} g(x) = -x^2 + 12x - 37 \\ \underline{+37} \qquad \qquad \underline{+37} \\ g(x) + 37 = -x^2 + 12x \\ g(x) + 37 - 36 = -(x^2 - 12x + 36) \\ \qquad \qquad \qquad \frac{1}{2}(-12) = -6 \\ \qquad \qquad \qquad (-6)^2 = 36 \\ \hline g(x) + 1 = -(x - 6)^2 \\ \underline{-1} \qquad \qquad \underline{-1} \\ g(x) = -(x - 6)^2 - 1 \end{array}$$

b) vertex (6, -1)

c) a.o.s. $x = 6$

d) $g(3) = -(3 - 6)^2 - 1$
 $g(3) = -10$



C. Rewrite in Vertex Form

Example 10Given $g(x) = 2x^2 - 8x + 17$,

- Write the function in vertex form.
- Give the coordinates of the vertex.
- Write the equation of the axis of symmetry.

$$\begin{aligned}
 \text{a)} \quad & \frac{g(x) = 2x^2 - 8x + 17}{-17} \\
 & \frac{g(x) - 17 = 2x^2 - 8x}{-17} \\
 & \frac{g(x) - 17 + 8 = 2(x^2 - 4x + 4)}{-17} \\
 & \quad \frac{1}{2}(-4) = -2 \\
 & \quad (-2)^2 = 4 \\
 & \frac{g(x) - 9 = 2(x - 2)^2}{+9} \\
 & \frac{g(x) = 2(x - 2)^2 + 9}{+9}
 \end{aligned}$$

b) vertex $(2, 9)$

c) a.o.s. $x = 2$

C. Rewrite in Vertex Form

Example 11Given $g(x) = 3x^2 + 9x - 1$,

- Write the function in vertex form.
- Give the coordinates of the vertex.
- Write the equation of the axis of symmetry.

$$\begin{aligned}
 \text{a)} \quad & \frac{g(x) = 3x^2 + 9x - 1}{+1} \\
 & \frac{g(x) + 1 = 3x^2 + 9x}{+1} \\
 & \frac{g(x) + 1 + \frac{9}{4} = 3(x^2 + 3x + \frac{9}{4})}{+1} \\
 & \quad \frac{1}{2}(3) = \frac{3}{2} \\
 & \quad (\frac{3}{2})^2 = \frac{9}{4} \\
 & \frac{g(x) + \frac{31}{4} = 3(x + \frac{3}{2})^2}{+1} \\
 & \frac{g(x) = 3(x + \frac{3}{2})^2 - \frac{31}{4}}{+1}
 \end{aligned}$$

b) vertex $(-\frac{3}{2}, -\frac{31}{4})$

c) a.o.s. $x = -\frac{3}{2}$

C. Rewrite in Vertex Form

Example 12

Given $g(x) = -\frac{1}{4}x^2 - 4x - 4$,

- a) Write the function in vertex form.
 b) Give the coordinates of the vertex.
 c) Write the equation of the axis of symmetry.

a) $g(x) = -\frac{1}{4}x^2 - 4x - 4$

b) vertex $(-8, 12)$

$g(x) + 4 = -\frac{1}{4}x^2 - 4x$

c) a.o.s. $x = -8$

$g(x) + 4 - 16 = -\frac{1}{4}(x^2 + 16x + 64)$

$\frac{1}{2}(16) = 8$

$(8)^2 = 64$

$g(x) - 12 = -\frac{1}{4}(x+8)^2$

$g(x) = -\frac{1}{4}(x+8)^2 + 12$

D. Find the minimum or maximum value

Example 13

Find the maximum or minimum value of the function $y = 3(x - 3)^2 - 4$. vertex $(3, -4)$



$\text{min} = -4 \text{ when } x = 3$

Example 14

Find the maximum or minimum value of the function $y = -15(x - 25)^2 + 130$.



vertex $(25, 130)$

$\text{max} = 130 \text{ when } x = 25$

1-1 Standard Form of Quadratic Functions.doc