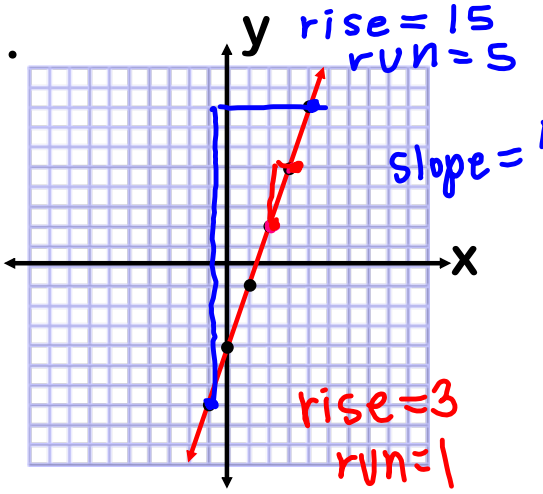
# 3.4 SLOPE & RATE OF CHANGE

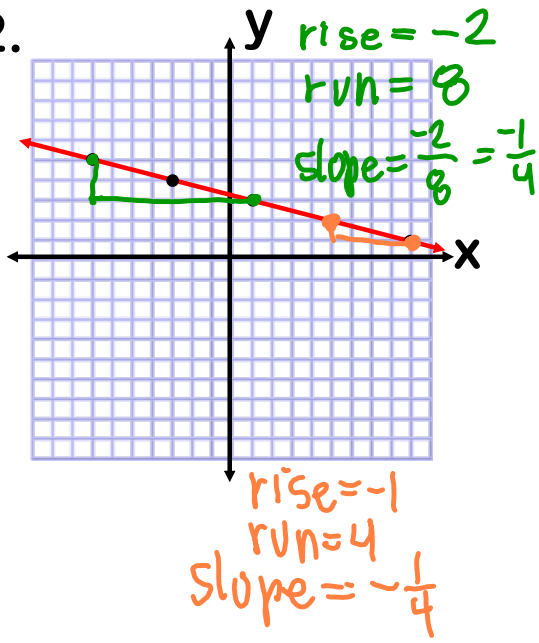
$$slope = \frac{rise}{run}$$

## Find the slope of the following lines.

1.

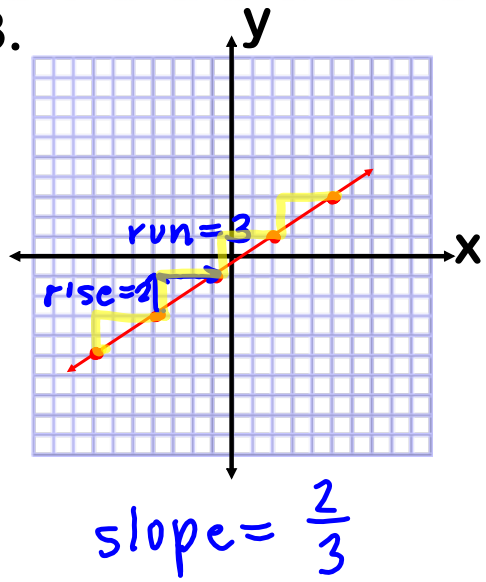



2.

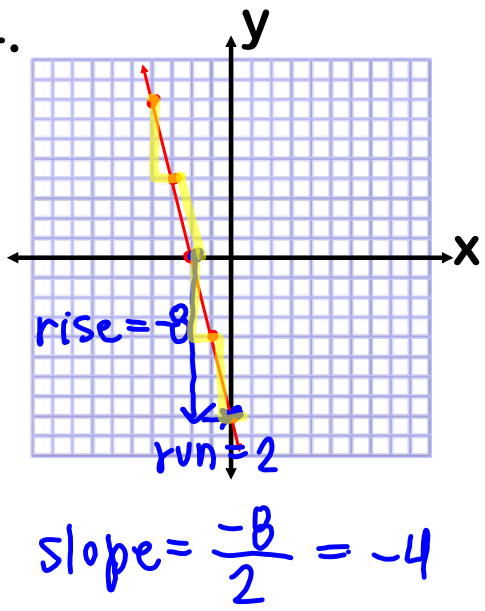



## Find the slope of the following lines.

3.

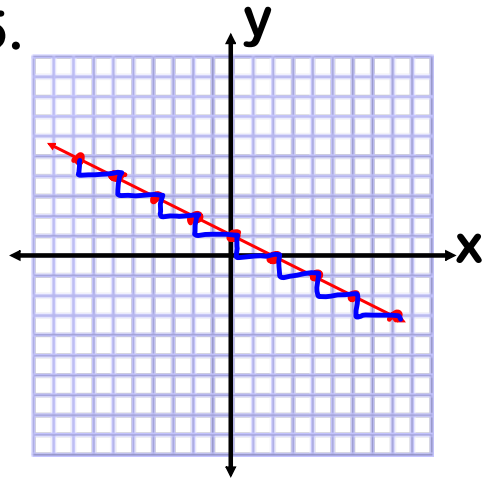



4.



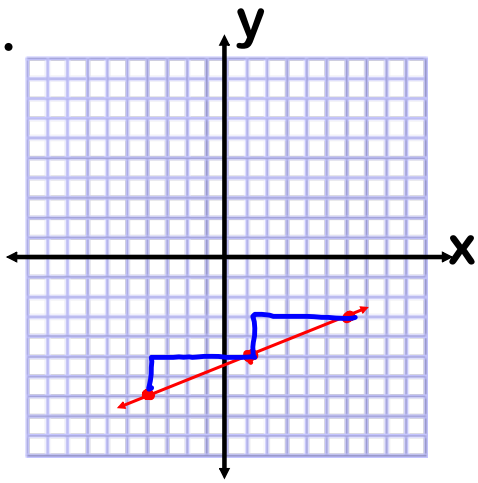
# Find the slope of the following lines.

5.



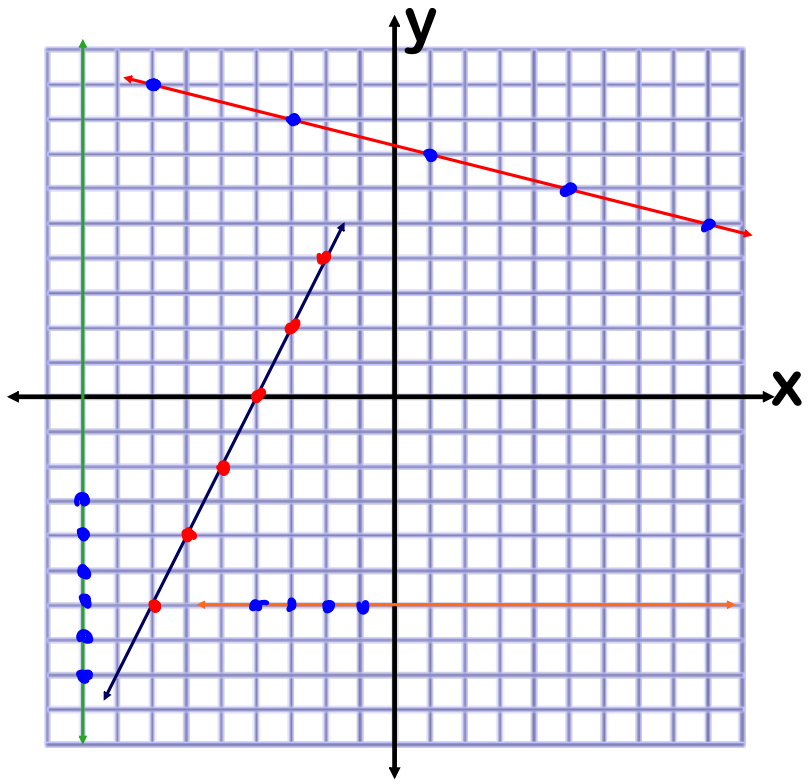

slope = $-\frac{1}{2}$

6.




slope = $\frac{2}{5}$

## 7. Find the slopes of each line.




**red line**

$-\frac{1}{4}$

**green line**

$\frac{1}{0}$ undefined

↓

vertical

**blue line**

$\frac{2}{1} = 2$

**orange line**

$\frac{0}{1} = 0$

↓

horizontal

$$slope = \frac{rise}{run} = \frac{change\ in\ y}{change\ in\ x} \quad \frac{\Delta y}{\Delta x}$$

**8. Determine the slope of the line containing the points with the coordinates listed in the tables below.**

**a)**

| x | -2 | 0 | 2 | 4 | 6 |
|---|---|---|---|---|---|
| y | -3 | 0 | 3 | 6 | 9 |

$\Delta x = 2$, $\Delta y = 3$, slope $= \frac{3}{2}$

**b)**

| x | -4 | -1 | 2 | 5 | 8 |
|---|---|---|---|---|---|
| y | -3 | -2 | -1 | 0 | 1 |

$\Delta x = 3$, $\Delta y = 1$, slope $= \frac{1}{3}$

The <u>slope</u> $m$ of a line that passes through the points $(x_1, y_1)$ and $(x_2, y_2)$ is

$$m = \frac{rise}{run} = \frac{change\ in\ y}{change\ in\ x} = \frac{y_2 - y_1}{x_2 - x_1}$$

**9. Find the slope of the line that passes through the points (1,0) and (3,4).**

$(1,0)$: $x_1\ y_1$ $\quad$ $(3,4)$: $x_2\ y_2$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-0}{3-1} = \frac{4}{2} \rightarrow \boxed{m = 2}$$

10. Find the slope of the line that passes through the points (1,2) and (5,2).

$(x_1, y_1)$ $(x_2, y_2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-2}{5-1} = \frac{0}{4} \rightarrow \boxed{m = 0}$$

11. Find the slope of the line that passes through the points (5,-1) and (5,3).

$(x_1, y_1)$ $(x_2, y_2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3+1}{5-5} = \frac{4}{0} \rightarrow \boxed{\text{undefined}}$$

12. Find the slope of the line that passes through the points (-2,1) and (1, -3).

$(x_1, y_1)$ $(x_2, y_2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-1}{1+2} \rightarrow \boxed{m = -\frac{4}{3}}$$

13. Find the slope of the line that passes through the points $(\frac{3}{4}, \frac{3}{4})$ and $(\frac{3}{8}, \frac{1}{2})$.

$(x_1, y_1)$ $(x_2, y_2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\frac{2}{4} - \frac{3}{4}}{\frac{3}{8} - \frac{6}{8}} = \frac{-\frac{1}{4}}{-\frac{3}{8}} = -\frac{1}{4} \cdot -\frac{2}{3}$$

$$\boxed{m = \frac{2}{3}}$$

14. Determine the value of r so the line through (r,4) and (9,-2) has a slope of $-\frac{3}{2}$.

$x_1\ y_1$ $x_2\ y_2$ ← m

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$-\frac{3}{2} = \frac{-2-4}{9-r}$$

$$\frac{-3}{2} = \frac{-6}{9-r}$$ ← proportion

$$-3(9-r) = (2)(-6)$$

$$-27 + 3r = -12$$

$$+27 \quad +27$$

$$\frac{3r}{3} = \frac{15}{3}$$

$$r = 5$$

15. Determine the value of r so the line through (-2,4) and (r,5) has a slope of $\frac{1}{5}$.

$x_1\ y_1$ $x_2\ y_2$ ← m

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{1}{5} = \frac{5-4}{r+2}$$

$$\frac{1}{5} = \frac{1}{r+2}$$

$$1(r+2) = 1 \cdot 5$$

$$r + 2 = 5$$

$$-2 \quad -2$$

$$r = 3$$

16. Determine the value of r so the line through (3,4) and (-1,r) has a slope of $-\frac{3}{4}$.

$x_1\ y_1$ $x_2\ y_2$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{-3}{4} = \frac{r-4}{-1-3}$

$\frac{-3}{4} = \frac{r-4}{-4}$

$4(r-4) = (-3)(-4)$

$4r - 16 = 12$

$+16 \quad +16$

$\frac{4r}{4} = \frac{28}{4}$

$r = 7$

17. Determine the value of r so the line through (4,r) and (r,2) has a slope of $-\frac{5}{3}$.

$x_1\ y_1$ $x_2\ y_2$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{-5}{3} = \frac{2-r}{r-4}$

$3(2-r) = -5(r-4)$

$6 - 3r = -5r + 20$

$+5r \quad +5r$

$6 + 2r = 20$

$-6 \quad -6$

$\frac{2r}{2} = \frac{14}{2}$

$r = 7$

# A rate of change compares a change in one quantity to a change in another quantity.

For example, if you are paid $60 for working 5 hours, then your hourly wage is $12 per hour, a rate of change that describes how your pay increases with respect to time spent working.

18. The table shows the cost of using a computer at an Internet cafe for a given amount of time. Find the rate of change in cost with respect to time. $\frac{\Delta y}{\Delta x}$ $\frac{\Delta C}{\Delta t}$

| | | | | |
|---|---|---|---|---|
| $x \rightarrow$ | Time (hours) | 2 | 4 | 6 |
| $y \rightarrow$ | Cost (dollars) | 7 | 14 | 21 |

$\Delta t = 2$

$\Delta C = 7$

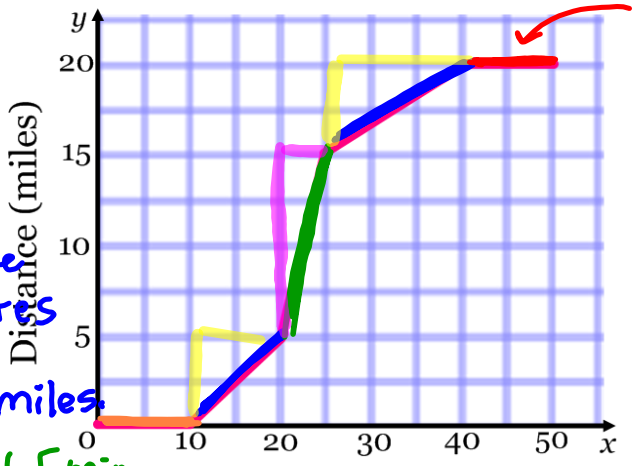
$$\frac{\Delta C}{\Delta t} = \frac{\$7}{2hr} = \$3.50/hr$$

# 19. The graph shows a ferry's trip across a a lake. Describe the trip in words.

For the first 10 min, they were loading & sitting at the dock.

Then over the next 10 minutes they went a distance of 5 miles.

Over the next 5min, they went a distance of 10 miles. (sped up)




For last 10 minutes, they didn't move (unloading).

Over next 15 minutes, they went a dist. of 10 miles. (slowed down)

| The Word Problem | In 2005 Mika made 49 meatballs for the annual school banquet. In 2007 Mika made 63 meatballs. Find the Rate of Change (i.e. the Slope) in **Meatballs** made by Mika between 2005 & 2007. | |
|---|---|---|
| Identify the **word** for the Independent Variable X & the Dependent Variable Y<br>Put just 1 **WORD** in each box! | Independent Variable X<br>Year | Dependent Variable Y<br>Meatballs |
| 2nd Point $(x_2, y_2)$<br>(the later, greater Indep value) | 2007 | 63 |
| 1st Point $(x_1, y_1)$<br>(the earlier, lesser Indep value) | 2005 | 49 |
| **Rise** is the Change in Dependent Y: $[y_2 - y_1]$ | 63 - 49 | |
| **Run** is the Change in Independent X: $[x_2 - x_1]$ | 2007 - 2005 | |
| Rate of Change or Slope is:<br>**Rise** over **Run**<br>Change in y / Change in x<br>Depend Change / Independ Change<br>(Original Ratio on Left & equivalent Unit Ratio on Right) | $\frac{14}{2}$<br>↓<br>7 | |
| Write **Unit** Rate of Change as<br>"An **increase** or A **decrease** of # ? **per** ?" | An *Increase* of 7 meatballs per year | |

| The Word Problem | There were 512 Bacteria on your Desk at the start of Science Class, 40 minutes later there were 2,048 Bacteria on your desk! Find the Rate of Change (i.e. the Slope) in **Bacteria** between the start and end of Science class. | |
|---|---|---|
| Identify the **word** for the Independent Variable X & the Dependent Variable Y<br>Put just 1 **WORD** in each box! | **Independent Variable X** | **Dependent Variable Y** |
| **2nd** Point $(x_2, y_2)$<br>(the later, greater Indep value) | | |
| **1st** Point $(x_1, y_1)$<br>(the earlier, lesser Indep value) | | |
| **Rise** is the Change in Dependent Y: $[y_2 - y_1]$ | | |
| **Run** is the Change in Independent X: $[x_2 - x_1]$ | | |
| Rate of Change or Slope is: **Rise** over **Run**<br>Change in y / Change in x<br>Depend Change / Independ Change<br>(Original Ratio on Left & equivalent Unit Ratio on Right) | | |
| Write **Unit** Rate of Change as "An **increase** or A **decrease** of # ? **per** ?" | | |

| The Word Problem | The average price of a prom dress in 1980 was $40. By 2010 the cost had risen to $130. Find the Rate of Change (i.e. the Slope) in the **Dollars** or cost of a prom dress between 1980 & 2010. | |
|---|---|---|
| Identify the **word** for the Independent Variable X & the Dependent Variable Y<br>Put just 1 **WORD** in each box! | **Independent Variable X** | **Dependent Variable Y** |
| **2nd** Point $(x_2, y_2)$<br>(the later, greater Indep value) | | |
| **1st** Point $(x_1, y_1)$<br>(the earlier, lesser Indep value) | | |
| **Rise** is the Change in Dependent Y: $[y_2 - y_1]$ | | |
| **Run** is the Change in Independent X: $[x_2 - x_1]$ | | |
| Rate of Change or Slope is: **Rise** over **Run**<br>Change in y / Change in x<br>Depend Change / Independ Change<br>(Original Ratio on Left & equivalent Unit Ratio on Right) | | |
| Write **Unit** Rate of Change as "An **increase** or A **decrease** of # ? **per** ?" | | |