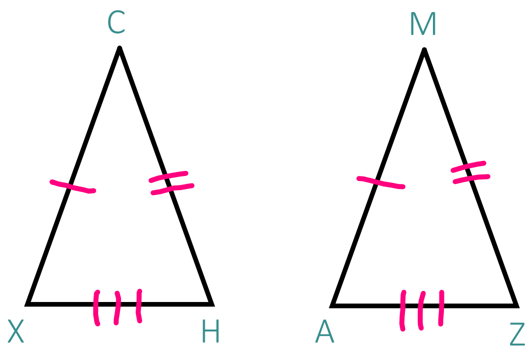
4.4 - 4.6 PROVING TRIANGLES CONGRUENT

Postulate 19: SSS Postulate Side-Side-Side

If the sides of one triangle are congruent to the sides of a second triangle, then the triangles are congruent.



$\Delta CXH \cong \Delta MAZ$
SSS Post.

Example 1

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$\Delta STU \cong \Delta XYZ$ by SSS

Given ΔSTU with vertices S(0, 5), T(0, 0), and U(-2, 0) and ΔXYZ with vertices X(4, 8), Y(4, 3), and Z(6, 3), determine if Δ STU ≅ ΔXYZ.

$ST=\sqrt{(0-0)^2+(0-5)^2}$
$=\sqrt{(0)^2+(-5)^2}$
$=\sqrt{0+25}$
$=\sqrt{25}=5$

$\cong$

$XY=\sqrt{(4-4)^2+(3-8)^2}$
$=\sqrt{(0)^2+(-5)^2}$
$=\sqrt{0+25}$
$=\sqrt{25}=5$

$TU=\sqrt{(-2-0)^2+(0-0)^2}$
$=\sqrt{(-2)^2+(0)^2}$
$=\sqrt{4+0}$
$=\sqrt{4}=2$

$\cong$

$YZ=\sqrt{(6-4)^2+(3-3)^2}$
$=\sqrt{(2)^2+(0)^2}$
$=\sqrt{4+0}$
$=\sqrt{4}=2$

$SU=\sqrt{(-2-0)^2+(0-5)^2}$
$=\sqrt{(-2)^2+(-5)^2}$
$=\sqrt{4+25}$
$=\sqrt{29}$

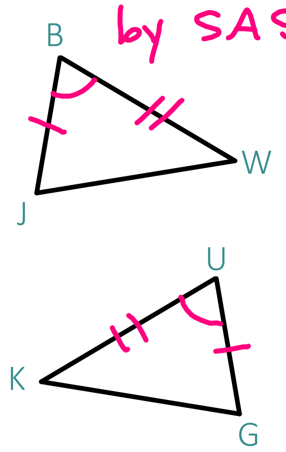
$\cong$

$XZ=\sqrt{(6-4)^2+(3-8)^2}$
$=\sqrt{(2)^2+(-5)^2}$
$=\sqrt{4+25}$
$=\sqrt{29}$

Postulate 20: SAS Postulate Side-Angle-Side

If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.

$\triangle BJW \cong \triangle UGK$ by SAS



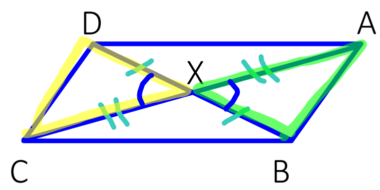
The included angle is the angle formed by two given sides.

Example 2

Write a proof for the following.

Given: X is the midpoint of $\overline{BD}$.
X is the midpoint of $\overline{AC}$.

Prove: $\Delta DXC \cong \Delta BXA$



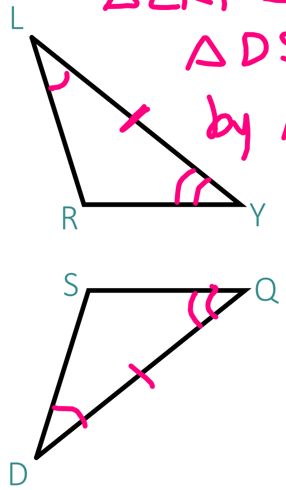
Statements	Reasons
① X is mdpt of $\overline{BD}$ & $\overline{AC}$	① given
② $\overline{XD} \cong \overline{XB}$; $\overline{XA} \cong \overline{XC}$	② def of mdpt
③ $\angle AXB \cong \angle CXD$	③ vert $\angle$s $\cong$
④ $\Delta DXC \cong \Delta BXA$	④ SAS

Postulate 21: ASA Postulate Angle-Side-Angle

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, the triangles are congruent.

$\triangle LRY \cong \triangle DSQ$ by ASA

The included side is the side of the triangle formed between two given angles.

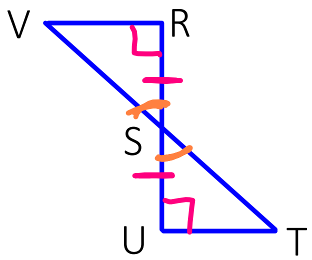


Example 3

Write a proof for the following.

Given: $\overline{VR} \perp \overline{RS}$, $\overline{UT} \perp \overline{SU}$, $\overline{RS} \cong \overline{US}$

Prove: $\triangle VRS \cong \triangle TUS$

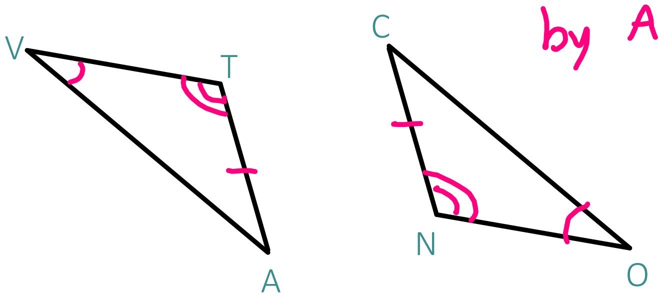


Statements	Reasons
① $\overline{VR} \perp \overline{RS}$, $\overline{UT} \perp \overline{SU}$, $\overline{RS} \cong \overline{US}$	① given
② $\angle R$ is a right $\angle$ $\angle U$ is a right $\angle$	② def of $\perp$
*③ $\angle R \cong \angle U$	③ all right $\angle$s $\cong$
④ $\angle VSR \cong \angle UST$	④ vert. $\angle$s $\cong$
⑤ $\triangle VRS \cong \triangle TUS$	⑤ ASA

Theorem 4-6: AAS Theorem Angle-Angle-Side

If two angles and a nonincluded side of a triangle are congruent to the corresponding two angles and a side of a second triangle, the two triangles are congruent.

$\triangle VTA \cong \triangle ONC$ by AAS



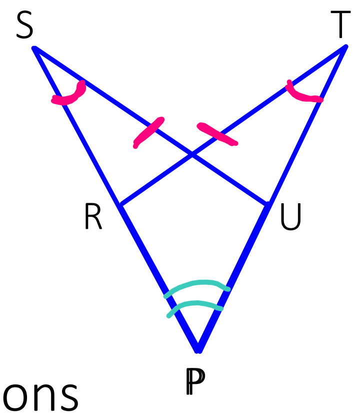
Example 4

Write a two-column proof.

Given: $\angle PSU \cong \angle PTR$

$\overline{SU} \cong \overline{TR}$

Prove: $\triangle SUP \cong \triangle TRP$



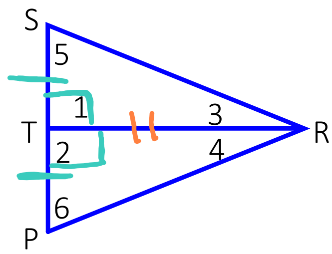
Statements	Reasons
① $\angle PSU \cong \angle PTR$, $\overline{SU} \cong \overline{TR}$	① given
② $\angle P \cong \angle P$	② reflexive prop.
③ $\triangle SUP \cong \triangle TRP$	③ AAS

Example 5

Write a two-column proof.

Given: $\angle 1$ and $\angle 2$ are right angles.
$\overline{ST} \cong \overline{TP}$

Prove: $\Delta STR \cong \Delta PTR$



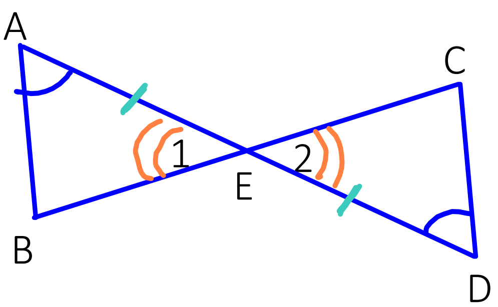
Statements	Reasons
① $\angle 1$ & $\angle 2$ are right $\angle$s; $\overline{ST} \cong \overline{TP}$	① given
② $\angle 1 \cong \angle 2$	② all right $\angle$s $\cong$
③ $\overline{TR} \cong \overline{TR}$	③ reflexive prop.
④ $\Delta STR \cong \Delta PTR$	④ SAS

Example 6

Write a two-column proof.

Given: $\overline{BE}$ bisects $\overline{AD}$.
$\angle A \cong \angle D$

Prove: $\Delta ABE \cong \Delta DEC$



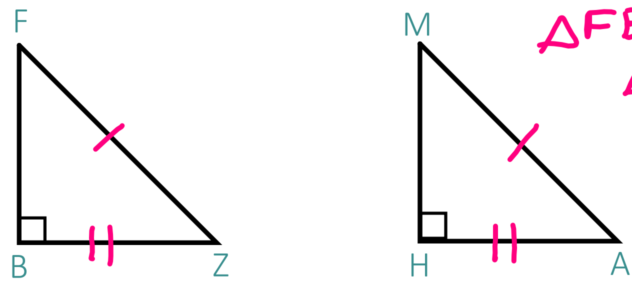
Statements	Reasons
① $\overline{BE}$ bisects $\overline{AD}$; $\angle A \cong \angle D$	① given
② $\overline{AE} \cong \overline{ED}$	② def of bisect
③ $\angle 1 \cong \angle 2$	③ vert $\angle$s are $\cong$
④ $\Delta ABE \cong \Delta DEC$	④ ASA

Theorem 4-8: HL Theorem

hypotenuse-leg

If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are congruent.

* first must prove that you have right Δ's



ΔFBZ ≅ ΔMHA by HL

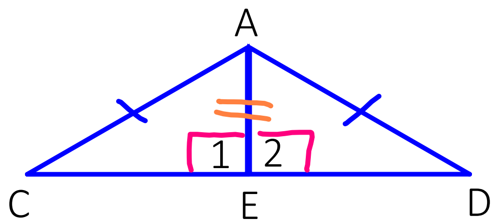
Example 7

Write a two-column proof.

Given: $\overline{AE} \perp \overline{CD}$

$\overline{AC} \cong \overline{AD}$

Prove: ΔACE ≅ ΔADE



Statements	Reasons
① $\overline{AE} \perp \overline{CD}$; $\overline{AC} \cong \overline{AD}$	① given
② ∠1 & ∠2 are right ∠s	② def of ⊥
③ ΔACE & ΔADE are right Δ's	③ def of right Δ
④ $\overline{AE} \cong \overline{AE}$	④ reflexive prop.
⑤ ΔACE ≅ ΔADE	⑤ HL