# 5.6 Operations with Complex Numbers

## Square Root of Negative Numbers

The square root of a negative real number has TWO imaginary roots : one positive, one negative.

$$\sqrt{-r} = \sqrt{-1} \cdot \sqrt{r} \quad \left(\text{where } \sqrt{-1} = i\right) = i\sqrt{r}$$

and

$$i^2 = i \cdot i = \sqrt{-1} \cdot \sqrt{-1} = -1$$

Examples : Simplify.

1. $\sqrt{-81} = 9i$

2. $\sqrt{-120} = 2i\sqrt{30}$

$120 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$

3. $\sqrt{-48} = 4i\sqrt{3}$

4. $\sqrt{-256} = 16i$

$48 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$

## Examples: Solve using the indicated method.

5. $x^2 + 6x + 10 = 0$

quadratic formula

$a=1 \quad b=6 \quad c=10$

$$x = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(10)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{-4}}{2}$$

$$x = \frac{-6 \pm 2i}{2}$$

$$x = \frac{-6}{2} \pm \frac{2i}{2}$$

$$\boxed{x = -3 \pm i}$$

$2\overline{)18}$
$9$

6. $-3x^2 - 10 = 44$

taking square roots

$$-3x^2 - 10 = 44$$
$$+10 \quad +10$$
$$\frac{-3x^2}{-3} = \frac{54}{-3}$$

$$\sqrt{x^2} = \sqrt{-18}$$

$$\boxed{x = \pm 3i\sqrt{2}}$$

## Examples: Solve using the indicated method.

7. $2(x-1)^2 + 12 = 0$

taking square roots

$$2(x-1)^2 + 12 = 0$$
$$-12 \quad -12$$
$$\frac{2(x-1)^2}{2} = \frac{-12}{2}$$

$$\sqrt{(x-1)^2} = \sqrt{-6}$$

$$x - 1 = \pm i\sqrt{6}$$
$$+1 \quad +1$$

$$\boxed{x = 1 \pm i\sqrt{6}}$$

8. $x^2 + 4x + 20 = 0$

completing the square

$$x^2 + 4x + 20 = 0$$
$$-20 \quad -20$$

$$x^2 + 4x \ \underline{+4} = -20 \ \underline{+4}$$

$\frac{1}{2}(4) = 2$

$(2)^2 = 4$

$$\sqrt{(x+2)^2} = \sqrt{-16}$$

$$x + 2 = \pm 4i$$
$$-2 \quad -2$$

$$\boxed{x = -2 \pm 4i}$$

Examples: Solve using any method.

9. $-\frac{1}{3}(x-7)^2+5=23$

$-5 \quad -5$

$-\frac{1}{3}(x-7)^2=18 \cdot -\frac{3}{1}$

$\sqrt{(x-7)^2}=\sqrt{-54}$

$2|54$
$3|27$
$3|9$
$3$

$x-7=\pm 3i\sqrt{6}$

$+7 \quad +7$

$x=7\pm 3i\sqrt{6}$

10. $3x^2-5x=-4$

$+4 \quad +4$

$3x^2-5x+4=0$

$a=3 \quad b=-5 \quad c=4$

$x=\frac{5\pm\sqrt{(-5)^2-4(3)(4)}}{2(3)}$

$x=\frac{5\pm\sqrt{-23}}{6}$

$x=\frac{5\pm i\sqrt{23}}{6}$

The standard form of a complex number is $a+bi$

real part

imaginary part

Every number can be written as a complex number.

$3+2i$ ⟶ imaginary number

$0+2i$ ⟶ pure imaginary number

$9+0i$ ⟶ real number

# Adding and Subtracting Complex Numbers

Add or subtract: real part to real part
imaginary part to imaginary part

Examples: Simplify.

11. (4 - i) + (3 + 2i)

$7 + i$

12. (7 - 5i) - (1 - 5i)

$7 - 5i - 1 + 5i$

$6$

13. 6 - (-2 + 9i) + (-8 + 4i)

$6 + 2 - 9i - 8 + 4i$

$-5i$

14. 2i - (3 + i) + (2 - 3i)

$2i - 3 - i + 2 - 3i$

$-1 - 2i$

$i^2 = -1$ Multiplying Complex Numbers

Examples: Simplify.

15. 5i (-2 + i)

$-10i + 5i^2$

$-10i + 5(-1)$

$-10i - 5 \longrightarrow -5 - 10i$

16. (-1 + 2i)(7 - 4i)

$-7 + 4i + 14i - 8i^2$

$-7 + 18i - 8(-1)$

$-7 + 18i + 8$

$1 + 18i$

17. (6 + 3i)(6 - 3i)

$36 - 18i + 18i - 9i^2$

$36 - 9(-1)$

$36 + 9$

$45$

18. $(2 + 5i)^2 \rightarrow (2 + 5i)(2 + 5i)$

$4 + 10i + 10i + 25i^2$

$4 + 20i + 25(-1)$

$4 + 20i - 25$

$-21 + 20i$