

Use completing the square to solve for x.

$$\frac{a}{1} \cdot \frac{b^2}{4a^2}$$

$$ax^2 + bx + c = 0$$

$$\frac{-c \cdot 4a^2}{4a^2} + \frac{ab^2}{4a^2}$$

$$\frac{-4a^2c}{4a^2} + \frac{ab^2}{4a^2}$$

$$\frac{ax^2 + bx}{a} = \frac{-c}{a} + \frac{ab^2}{4a^2}$$

$$a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) = -c + \frac{ab^2}{4a^2}$$

$$\frac{1}{2}\left(\frac{b}{a}\right) = \frac{b}{2a}$$

$$\left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$$

$$\frac{1}{a} a\left(x + \frac{b}{2a}\right)^2 = \frac{-4a^2c + ab^2}{4a^2} \cdot \frac{1}{a}$$

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{-4ac + b^2}{4a^2}}$$

$$x + \frac{b}{2a} = \frac{\pm\sqrt{-4ac + b^2}}{2a}$$

$$-\frac{b}{2a} \quad -\frac{b}{2a}$$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{-4ac + b^2}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



5.5 Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

To use the quadratic formula:

1. Equation must be in the form $ax^2 + bx + c = 0$.
2. Plug the values of a, b, and c into the formula.
3. Simplify the radical.
4. Simplify the answer.



Solve using the quadratic formula: $2x^2 - 3x - 2 = 0$

$$a=2 \quad b=-3 \quad c=-2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(2)(-2)}}{2(2)}$$

$$x = \frac{3 \pm \sqrt{9 + 16}}{4}$$

$$x = \frac{3 \pm \sqrt{25}}{4}$$

$$\frac{3+5}{4} = \frac{8}{4} = \boxed{2 = x}$$

$$x = \frac{3 \pm 5}{4}$$

$$\frac{3-5}{4} = \frac{-2}{4} = \boxed{-\frac{1}{2} = x}$$

Solve using the quadratic formula: $4x^2 = 12x - 9$

$$\begin{aligned} & \frac{-4x^2 \quad -4x^2}{0 = -4x^2 + 12x - 9} \\ & a = -4 \quad b = 12 \quad c = -9 \end{aligned}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-12 \pm \sqrt{(12)^2 - 4(-4)(-9)}}{2(-4)}$$

$$x = \frac{-12 \pm \sqrt{0}}{-8}$$

$$x = \frac{-12 \pm 0}{-8}$$

$$\frac{-12+0}{-8}$$

$$\frac{-12}{-8} = \boxed{\frac{3}{2} = x}$$

$$\frac{-12-0}{-8}$$

Solve using the quadratic formula: $3x^2 + 2x = 3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(3)(-3)}}{2(3)}$$

$$x = \frac{-2 \pm \sqrt{40}}{6}$$

$$x = \frac{-2 \pm 2\sqrt{10}}{6}$$

$$x = \frac{2(-1 \pm \sqrt{10})}{6}$$

$$x = \frac{-1 \pm \sqrt{10}}{3}$$

$$\begin{array}{r} 3x^2 + 2x = 3 \\ \underline{-3 \quad -3} \\ 3x^2 + 2x - 3 = 0 \\ a=3 \quad b=2 \quad c=-3 \end{array}$$

$$\begin{array}{r} 2 \overline{)40} \\ \underline{20} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

Solve using the quadratic formula: $5x^2 + 1 = 7x$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4(5)(1)}}{2(5)}$$

$$x = \frac{7 \pm \sqrt{29}}{10}$$

$$\begin{array}{r} 5x^2 + 1 = 7x \\ \underline{-7x \quad -7x} \\ 5x^2 - 7x + 1 = 0 \\ a=5 \quad b=-7 \quad c=1 \end{array}$$



DISCRIMINANT: $b^2 - 4ac$

From the discriminant, we can determine:

1. the number of roots or solutions (how many? 1 or 2?)
2. the type of roots or solutions (real or imaginary?)

If the value of the discriminant is:	Then the roots or solutions are:
1. 0	1. one real
2. positive	2. two real
3. negative	3. two imaginary

Without solving, find the value of the discriminant and describe the roots (number and type).

1. $x^2 - 3x - 40 = 0$ $b^2 - 4ac = (-3)^2 - 4(1)(-40)$
 $= 9 + 160$
 $= 169$
 2 real solutions

2. $3x^2 + 9x - 2 = 0$ $b^2 - 4ac = (9)^2 - 4(3)(-2)$
 $= 81 + 24$
 $= 105$
 2 real solutions

3. $7x^2 + 6x + 2 = 0$ $b^2 - 4ac = (6)^2 - 4(7)(2)$
 $= 36 - 56$
 $= -20$
 2 imaginary solutions

4. $x^2 - \frac{1}{2}x + \frac{1}{16} = 0$ $b^2 - 4ac = (-\frac{1}{2})^2 - 4(1)(\frac{1}{16})$
 $= \frac{1}{4} - \frac{1}{4}$
 $= 0$
 1 real solution

MODELING LAUNCHED OBJECTS

Models an object being dropped:

$$h = -16t^2 + h_0$$

Models an object being launched or thrown:

$$h = -16t^2 + v_0t + h_0$$



v_0 = initial velocity (ft/sec)

h_0 = initial height (ft)

t = time (sec)

A juggler tosses a ball into the air. The ball leaves the juggler's hand 7 feet above the ground and has an initial velocity of 8 feet per second.

The juggler catches the ball when it falls back to a height of 4 feet. How long is the ball in the air?

$$v_0 = 8 \text{ ft/s} \quad h_0 = 7 \quad h = 4$$

$$h = -16t^2 + v_0t + h_0$$

$$4 = -16t^2 + 8t + 7$$

$$-4 = -16t^2 + 8t + 7$$

$$0 = -16t^2 + 8t + 3$$

$$a = -16 \quad b = 8 \quad c = 3$$

$$t = \frac{-8 \pm \sqrt{(8)^2 - 4(-16)(3)}}{2(-16)}$$

$$t = \frac{1}{4}$$

$$t = \frac{3}{4} \text{ sec}$$



Summary of Solving Quadratic Equations		
Method	When to Use	Example
Factoring	$c = 0$ or the expression is easily factorable.	$x^2 + 4x + 3 = 0$
Square Roots	The variable side of the equation is a perfect square.	$(x - 5)^2 = 24$
Completing the Square	$a = 1$ and b is an even number.	$x^2 + 6x - 10 = 0$
Quadratic Formula	Numbers are large or complicated, and the expression does not factor or does not factor easily.	$5x^2 - 7x - 8 = 0$