Use completing the square to solve for x.

$$ax^2 + bx + c = 0$$

$$-c \quad -c$$

$$ax^2 + bx = -c$$

$$a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) = -c + \frac{ab^2}{4a^2}$$

$$\frac{a}{1} \cdot \frac{b^2}{4a^2}$$

$$\frac{1}{2}\left(\frac{b}{a}\right) = \frac{b}{2a}$$

$$\left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$$

$$\frac{-c}{1} \cdot \frac{4a^2}{4a^2} + \frac{ab}{4a^2}$$

$$\frac{-4a^2c}{4a^2} + \frac{ab}{4a^2}$$

$$\frac{1}{a} \cdot a\left(x + \frac{b}{2a}\right)^2 = \frac{-4a^2c + ab}{4a^2} \cdot \frac{1}{a} \qquad a(-4ac + b)$$

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{-4ac + b^2}{4a^2}}$$

$$x + \frac{b}{2a} = \frac{\pm\sqrt{-4ac + b^2}}{2a}$$

$$-\frac{b}{2a} \qquad -\frac{b}{2a}$$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{-4ac + b^2}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

5.5 Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

To use the quadratic formula:

1. Equation must be in the form $ax^2 + bx + c = 0$.
2. Plug the values of a, b, and c into the formula.
3. Simplify the radical.
4. Simplify the answer.

Solve using the quadratic formula: $2x^2 - 3x - 2 = 0$

$a=2 \quad b=-3 \quad c=-2$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{3 \pm \sqrt{(-3)^2-4(2)(-2)}}{2(2)}$$

$$x = \frac{3 \pm \sqrt{9+16}}{4}$$

$$x = \frac{3 \pm \sqrt{25}}{4}$$

$$x = \frac{3 \pm 5}{4}$$

$$\frac{3+5}{4} = \frac{8}{4} = \boxed{2 = x}$$

$$\frac{3-5}{4} = \frac{-2}{4} = \boxed{-\frac{1}{2} = x}$$

Solve using the quadratic formula: $4x^2 = 12x - 9$

$$\begin{aligned} -4x^2 \quad & -4x^2 \\ 0 &= -4x^2+12x-9 \end{aligned}$$

$a=-4 \quad b=12 \quad c=-9$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-12 \pm \sqrt{(12)^2-4(-4)(-9)}}{2(-4)}$$

$$x = \frac{-12 \pm \sqrt{0}}{-8}$$

$$x = \frac{-12 \pm 0}{-8}$$

$$\frac{-12+0}{-8} \qquad \frac{-12-0}{-8}$$

$$\frac{-12}{-8} = \boxed{\frac{3}{2} = x}$$

Solve using the quadratic formula: $3x^2 + 2x = 3$

$$-3 \qquad -3$$

$$3x^2 + 2x - 3 = 0$$

$$a = 3 \quad b = 2 \quad c = -3$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(3)(-3)}}{2(3)}$$

$$x = \frac{-2 \pm \sqrt{40}}{6}$$

2 | 40
2 | 20
2 | 10
5

$$x = \frac{-2 \pm 2\sqrt{10}}{6}$$

$$x = \frac{2(-1 \pm \sqrt{10})}{6} \quad (6 \to 3)$$

$$\boxed{x = \frac{-1 \pm \sqrt{10}}{3}}$$

Solve using the quadratic formula: $5x^2 + 1 = 7x$

$$-7x \qquad -7x$$

$$5x^2 - 7x + 1 = 0$$

$$a = 5 \quad b = -7 \quad c = 1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4(5)(1)}}{2(5)}$$

$$\boxed{x = \frac{7 \pm \sqrt{29}}{10}}$$

DISCRIMINANT: $b^2 - 4ac$

From the discriminant, we can determine:
1. the number of roots or solutions (how many? 1 or 2?)
2. the type of roots or solutions (real or imaginary?)

If the value of the discriminant is:	Then the roots or solutions are:
1. 0	1. one real
2. positive	2. two real
3. negative	3. two imaginary

Without solving, find the value of the discriminant and describe the roots (number and type).

1. $x^2 - 3x - 40 = 0$ $b^2-4ac = (-3)^2 - 4(1)(-40)$
   $= 9 + 160$
   $= 169$
   2 real solutions

2. $3x^2 + 9x - 2 = 0$ $b^2-4ac = (9)^2 - 4(3)(-2)$
   $= 81 + 24$
   $= 105$
   2 real solutions

3. $7x^2 + 6x + 2 = 0$ $b^2-4ac = (6)^2 - 4(7)(2)$
   $= 36 - 56$
   $= -20$
   2 imaginary solutions

4. $x^2 - \frac{1}{2}x + \frac{1}{16} = 0$ $b^2-4ac = (-\frac{1}{2})^2 - 4(1)(\frac{1}{16})$
   $= \frac{1}{4} - \frac{1}{4}$
   $= 0$
   1 real solution