

2.6 Combining Functions

Sum of f and g : $(f + g)(x) = f(x) + g(x)$

Difference of f and g : $(f - g)(x) = f(x) - g(x)$

Product of f and g : $(f \cdot g)(x) = f(x) \cdot g(x)$

Quotient of f and g : $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, provided $g(x) \neq 0$
domain restriction

Examples

1. Find $(f + g)(0)$. = 5

$$\begin{array}{r} f(0) + g(0) \\ 5 + 0 \end{array}$$

2. Find $(f + g)(3)$. = 8

$$\begin{array}{r} f(3) + g(3) \\ 4 + 4 \end{array}$$

3. For what values of x is

$$(f - g)(x) = 0? \quad x = -3, 3$$

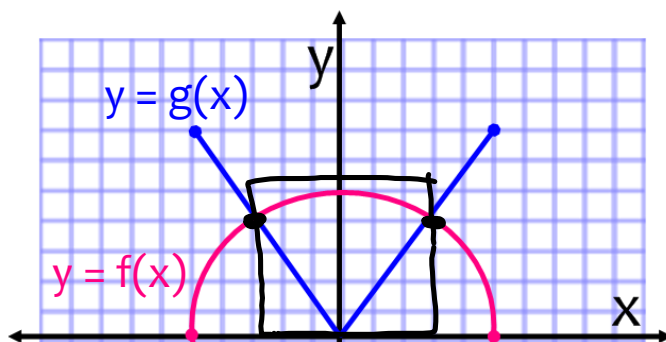
$$f(x) - g(x) = 0 \rightarrow f(x) = g(x)$$

4. For what values of x is $-3 < x < 3 \rightarrow (-3, 3)$

$$(f - g)(x) > 0?$$

$$f(x) - g(x) > 0$$

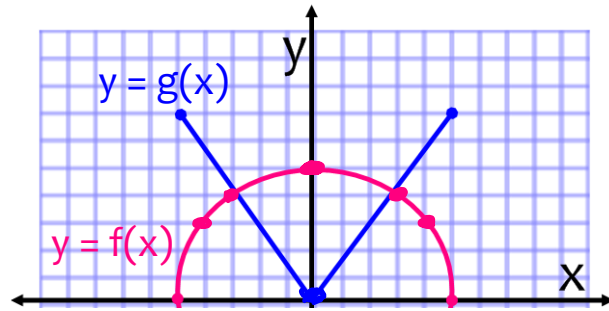
bigger smaller



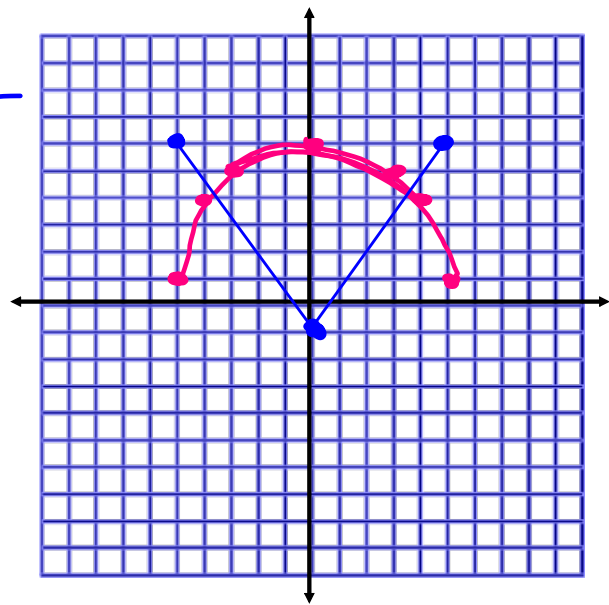
pink graph > blue graph

Examples

5. Graph $f(x) + 1$.
vert. shift 1 up



6. Graph $g(x) - 1$.
vert. shift down 1

Examples

Let $f(x) = x + 1$ and $g(x) = x^2 - 1$. Find each of the following functions and their domains.

7. $(f + g)(x)$

$$(x+1) + (x^2-1)$$

$$f+g = x^2 + x$$

$$D: (-\infty, \infty)$$

8. $(f - g)(x)$

$$(x+1) - (x^2-1)$$

$$x+1 - x^2 + 1$$

$$f-g = -x^2 + x + 2$$

$$D: (-\infty, \infty)$$

9. $(f \cdot g)(x)$

$$(x+1)(x^2-1)$$

$$f \cdot g = x^3 - x + x^2 - 1$$

$$D: (-\infty, \infty)$$

10. $\left(\frac{f}{g}\right)(x)$

$$\frac{x+1}{x^2-1} = \frac{x+1}{(x+1)(x-1)}$$

$$\frac{f}{g} = \frac{1}{x-1}$$

$$D: (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$$

Examples

Let $f(x) = x - 4$ and $g(x) = \sqrt{x - 4}$. Find each of the following functions and their domains.

11. $(f + g)(x)$
 $f + g = (x - 4) + \sqrt{x - 4}$
 $x - 4 \geq 0$
 $x \geq 4$
 $D: [4, \infty)$

12. $(f - g)(x)$
 $f - g = x - 4 - \sqrt{x - 4}$
 $x - 4 \geq 0$
 $D: [4, \infty)$

13. $(f \cdot g)(x)$
 $f \cdot g = (x - 4)\sqrt{x - 4}$
 $x - 4 \geq 0$
 $D: [4, \infty)$

14. $\left(\frac{f}{g}\right)(x) = \frac{x - 4}{\sqrt{x - 4}}$
 $x - 4 > 0$
 $D: (4, \infty)$

Examples

Let $f(x) = \frac{1}{x - 2}$ and $g(x) = \sqrt{x}$. Find each of the following functions and their domains.

15. $(f + g)(x)$
 $f + g = \frac{1}{x - 2} + \sqrt{x}$
 $x - 2 \neq 0$
 $x \neq 2$
 $x \geq 0$
 $D: [0, 2) \cup (2, \infty)$

16. $(f - g)(x)$
 $f - g = \frac{1}{x - 2} - \sqrt{x}$
 $D: [0, 2) \cup (2, \infty)$

17. $(f \cdot g)(x)$
 $f \cdot g = \frac{1}{x - 2} \cdot \sqrt{x}$
 $f \cdot g = \frac{\sqrt{x}}{x - 2}$
 $D: [0, 2) \cup (2, \infty)$

18. $\left(\frac{f}{g}\right)(x) = \frac{1}{x - 2} \div \sqrt{x}$
 $x > 0$
 $= \frac{1}{x - 2} \cdot \frac{1}{\sqrt{x}}$
 $= \frac{1}{\sqrt{x}(x - 2)}$
 $D: (0, 2) \cup (2, \infty)$

Composition of Functions

$$\underbrace{f \circ g}_{\text{f of g}} = (f \circ g)(x) = f(g(x))$$

Example

Let $f(x) = \frac{1}{x^2}$ and $g(x) = \sqrt{x-2}$.

19. Find a rule for $(f \circ g)(x)$ and give the domain of the composite function.

$$f \circ g = \underset{\substack{\uparrow \\ \text{outside}}}{f}(\underset{\substack{\uparrow \\ \text{inside}}}{g(x)}) = \frac{1}{(\sqrt{x-2})^2} = \frac{1}{x-2}$$

$x-2 > 0$
 $x > 2$

$D: (2, \infty)$

Examples

Let $f(x) = \frac{1}{x}$ and $g(x) = x + 1$.

20. Find rules for $(f \circ g)(x)$ and $(g \circ f)(x)$ and give the domain of each composite function.

$$f(g(x)) = \frac{1}{(x+1)} = \frac{1}{x+1} \quad D: (-\infty, -1) \cup (-1, \infty)$$

$x+1 \neq 0 \rightarrow x \neq -1$

$$g(f(x)) = \left(\frac{1}{x}\right) + 1 = \frac{1}{x} + \frac{x}{x} = \frac{x+1}{x} \quad x \neq 0$$

$D: (-\infty, 0) \cup (0, \infty)$

21. Find $(f \circ g)(4)$.

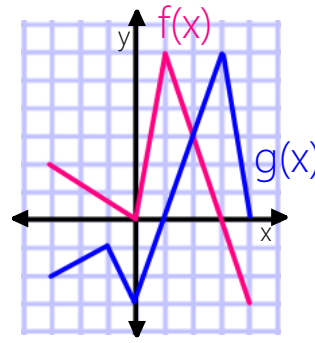
$$f(g(4)) = \frac{1}{4+1} = \frac{1}{5}$$

22. Find $(g \circ f)(-5)$.

$$g(f(-5)) = \frac{-5+1}{-5} = \frac{-4}{-5} = \frac{4}{5}$$

Examples

Use the given graphs of f and g to evaluate each expression below.



23. Find $(f \circ g)(2) = 0$

$$f(g(2))$$

$$f(3) = 0$$

25. Find $(f \circ g)(-2) = 1$

$$f(g(-2))$$

$$f(-\frac{1}{2}) = 1$$

24. Find $(g \circ f)(-3) = 3$

$$g(f(-3))$$

$$g(2) = 3$$

26. Find $(g \circ f)(3) = -3$

$$g(f(3))$$

$$g(0) = -3$$

Examples

Let $f(x) = \frac{x}{x+1}$, $g(x) = x^{10}$, and $h(x) = x + 3$.

27. Find $f \circ g \circ h$.

$$f(g(h(x)))$$

$$g(h(x)) = (x+3)^{10}$$

$$f(g(h(x))) = \frac{(x+3)^{10}}{(x+3)^{10} + 1}$$

28. Find $g \circ h \circ f$.

$$g(h(f(x)))$$

$$\begin{aligned} h(f(x)) &= \left(\frac{x}{x+1}\right) + \frac{3(x+1)}{x+1} \\ &= \frac{x}{x+1} + \frac{3x+3}{x+1} \\ &= \frac{4x+3}{x+1} \end{aligned}$$

$$g(h(f(x))) = \left(\frac{4x+3}{x+1}\right)^{10}$$

Examples

29. Given $F(x) = \sqrt[4]{x+9}$, find functions for f and g such that $F = f \circ g$. ← inside

$$f(x) = \sqrt[4]{x}$$

$$g(x) = x+9$$

30. Given $H(x) = |1 - x^3|$, find functions for f and g such that $H = f \circ g$. ← inside

$$f(x) = |x|$$

$$g(x) = 1 - x^3$$