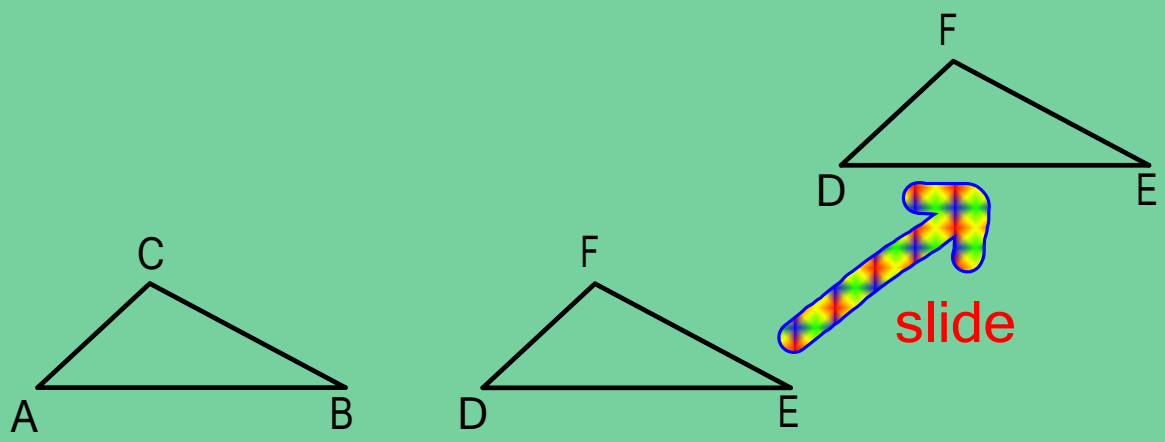
4.2 CONGRUENCE AND TRIANGLES

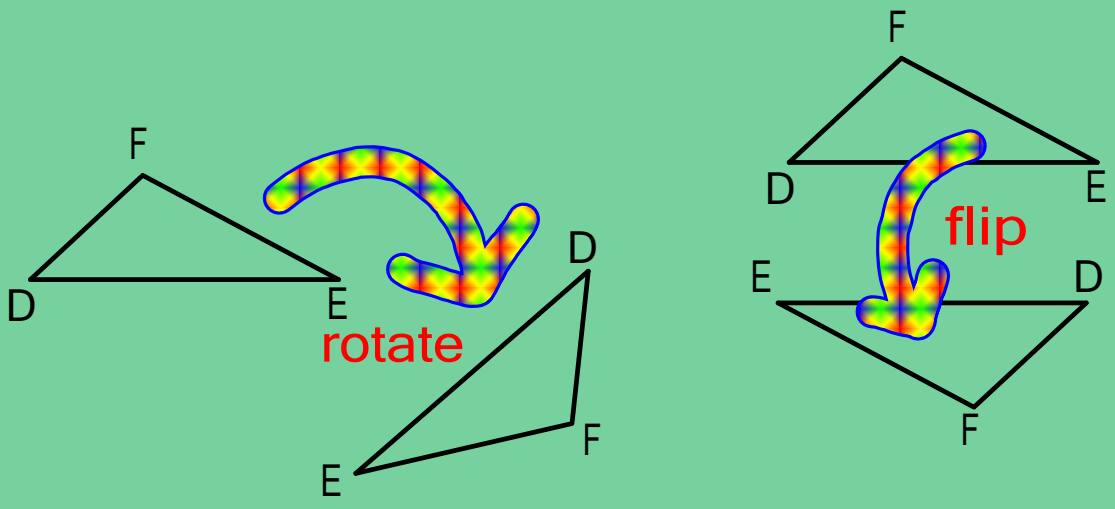
Congruent triangles are triangles that have the same shape and size.

Each triangle has six parts, three angles and three sides. If the corresponding six parts of one triangle are congruent to the six parts of another triangle, then the triangles are congruent .

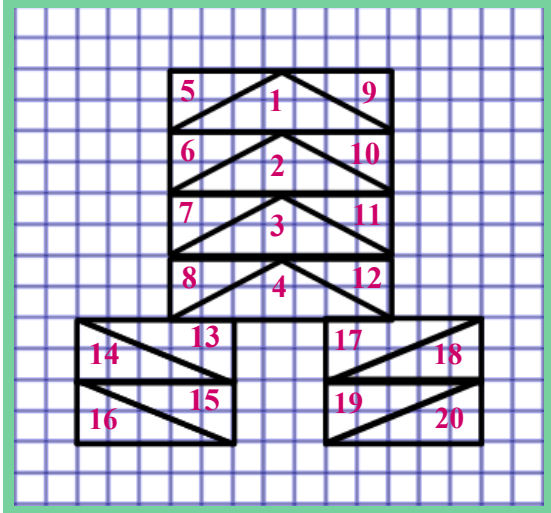
In the figure below, ΔDEF is congruent to ΔABC. If you slide ΔDEF up and to the right, ΔDEF is still congruent to ΔABC.



Also, if you rotate ΔDEF,
ΔDEF remains congruent to ΔABC.
If you flip ΔDEF,ΔDEF remains congruent.



If you slide, rotate, or flip a figure, congruence will not change.
These three transformations are examples of rigid motion (a transformation that preserves length, angle measure, and area).
These are also called congruence transformations because they change the position of a figure without changing its size or shape .



Example 1

A design for a quilt has been drawn on graph paper.

Identify the transformation from:

a) 5 to 9? reflection

b) 6 to 8? translation

c) 1 to 4? translation

d) 16 to 18? reflection & translation

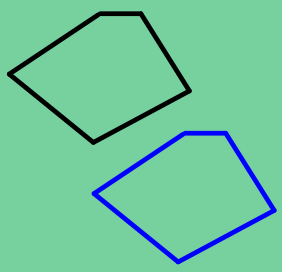
e) 10 to 7? reflection & translation

Example 2

Identify each transformation.

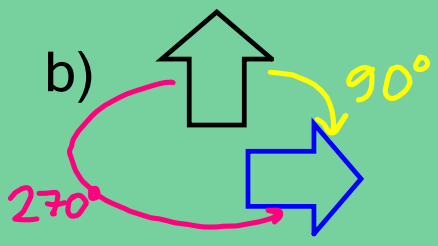
Which is not an example of rigid motion?

a)



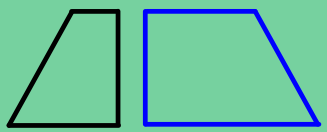
translation

b)



rotation

c)



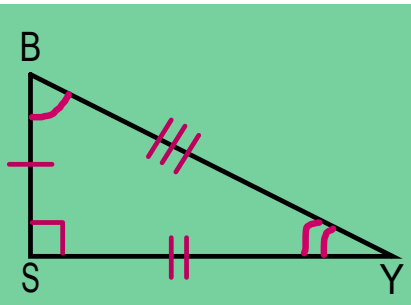
not rigid motion

d)

reflection

Example 3

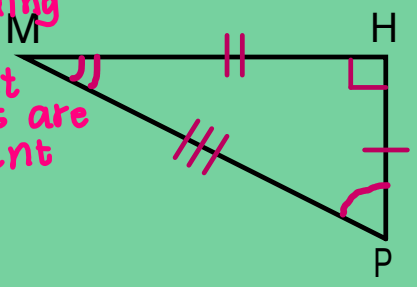
Refer to the figure at the right. Complete the congruence statement below:



$\Delta YSB \cong \Delta MHP$

2 arcs, 1 arc, right $\angle$s

Corresponding Parts of Congruent Triangles are Congruent

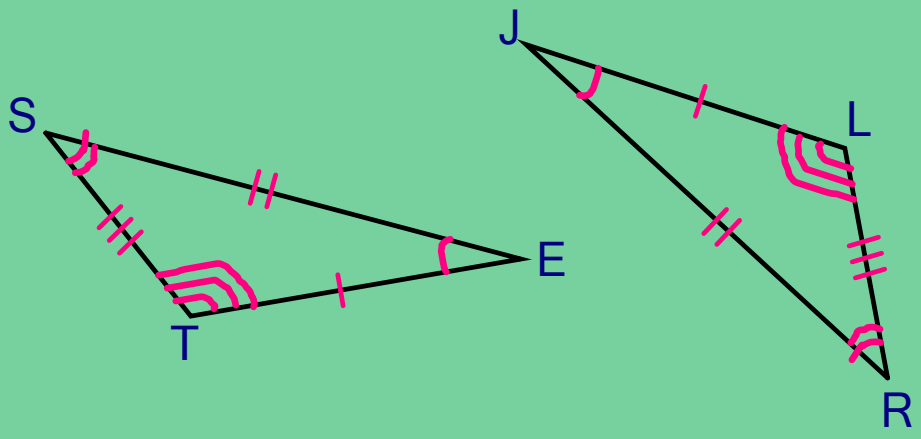


Definition of Congruent Triangles (CPCTC)

Two triangles are congruent if and only if their corresponding parts are congruent.

Example 4

$\Delta TSE \cong \Delta LRJ$



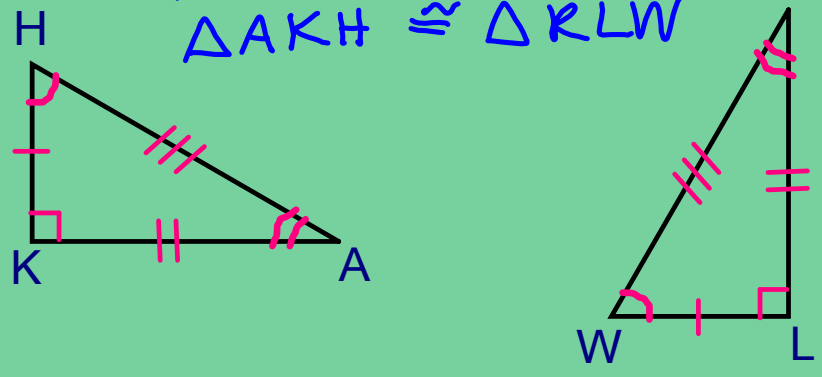
Example 5

$\Delta HKA \cong \Delta WLR$

$\Delta KHA \cong \Delta LWR$

$\Delta AHK \cong \Delta RWL$

$\Delta AKH \cong \Delta RLW$



Example 6

If $\Delta GHI \cong \Delta RST$, name all the corresponding angles and corresponding sides. Draw a figure showing the two triangles, and mark the corresponding parts.

angles

$\angle G \cong \angle R$

$\angle H \cong \angle S$

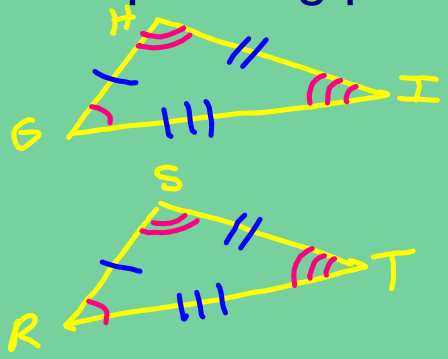
$\angle I \cong \angle T$

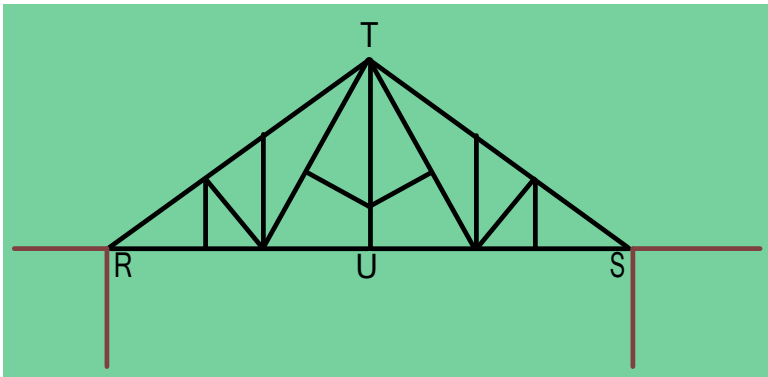
sides

$\overline{GH} \cong \overline{RS}$

$\overline{HI} \cong \overline{ST}$

$\overline{GI} \cong \overline{RT}$





Example 7

The bridge uses a simple triangular truss design. $\Delta TRU \cong TSU$. Name the corresponding congruent angles and sides.

angles
$\angle R \cong \angle S$
$\angle RTU \cong \angle STU$
$\angle TUR \cong \angle TUS$

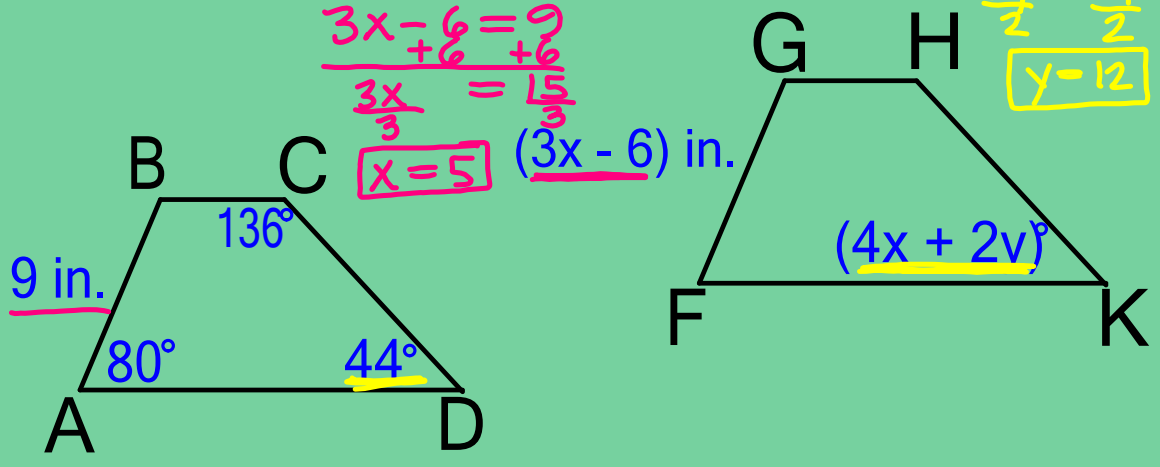
sides
$\overline{TR} \cong \overline{TS}$
$\overline{RU} \cong \overline{SU}$
$\overline{TU} \cong \overline{TU}$

Example 8

In the diagram, ABCD $\cong$ FGHK. Find the value of x and y.

$3x - 6 = 9$
$+6 \quad +6$
$\frac{3x}{3} = \frac{15}{3}$
$x = 5$

$4x + 2y = 44$
$4(5) + 2y = 44$
$20 + 2y = 44$
$-20 \quad -20$
$\frac{2y}{2} = \frac{24}{2}$
$y = 12$

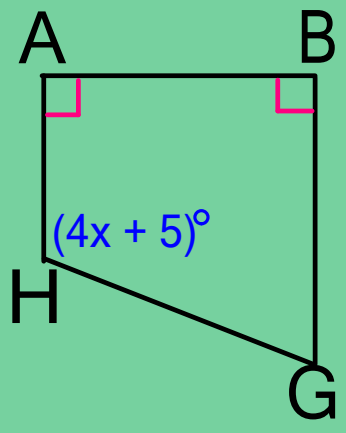


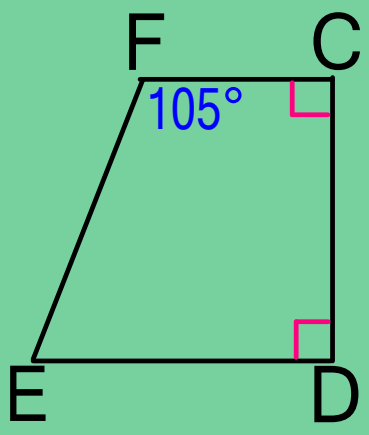
Example 9

In the diagram, ABGH $\cong$ CDEF.
Find the value of x and m $\angle$H.

$4x+5=105$
$-5 \quad -5$
$\frac{4x}{4} = \frac{100}{4}$
$x=25$

$m\angle H = 105°$





Theorem 4.3 Third Angles Theorem

If 2 angles of one triangle are congruent to 2 angles of another triangle, then the third angles are congruent.

$\triangle ABC \cong \triangle ABC$

Theorem 4.4

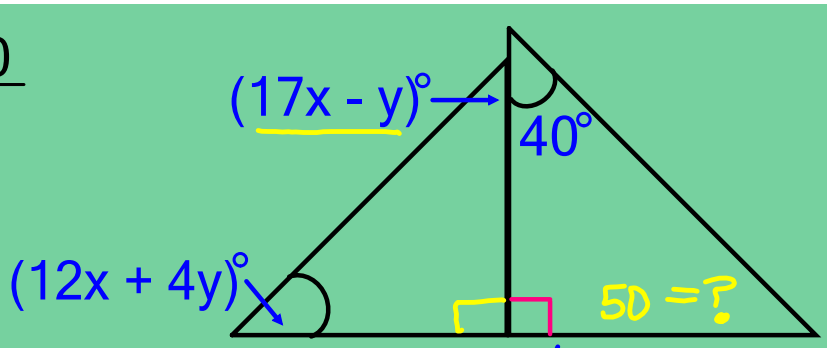
If $\triangle ABC \cong \triangle DEF$, then $\triangle DEF \cong \triangle ABC$.

Congruence of triangles is reflexive, symmetric, and transitive.

If $\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle GHI$,
then $\triangle ABC \cong \triangle GHI$.

Example 10

Find the value of x and y.



$12x + 4y = 40$

$4(17x - y) = (50)4$ →

$12x + 4y = 40$

$+\ 68x - 4y = 200$

$\frac{80x}{80} = \frac{240}{80}$

$x = 3$

$12(3) + 4y = 40$

$36 + 4y = 40$

$-36 \quad -36$

$\frac{4y}{4} = \frac{4}{4}$

$y = 1$