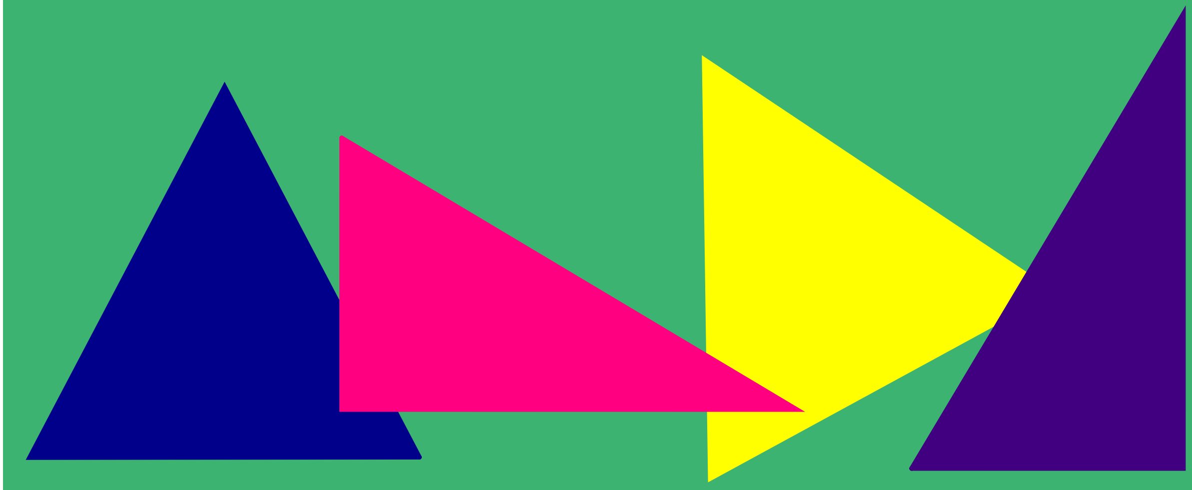
# 4.1 TRIANGLES AND ANGLES



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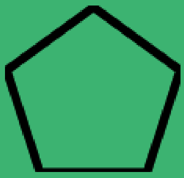
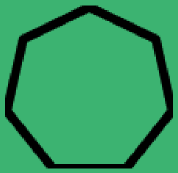
polygon - a closed figure in a plane that is ma
up of segments, called *sides*, that intersect only
at their endpoints, called *vertices*

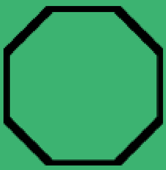


pentagon

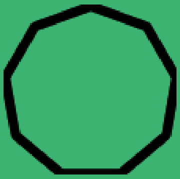
hexagon



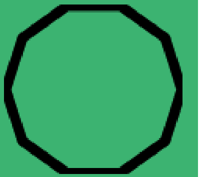
heptagon



octagon



nonagon



decagon

Can you name these?

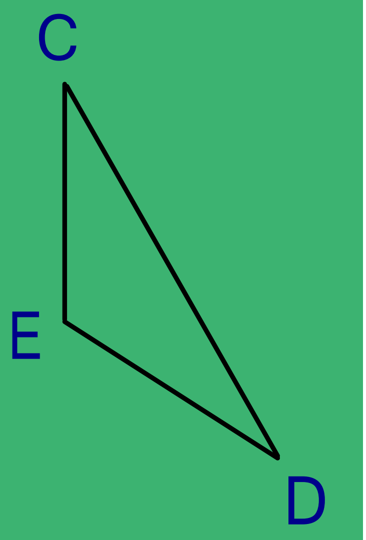
## triangle a three-sided polygon

Triangle CDE, written ∆CDE,
has the following parts:

SIDES: $\overline{CE}$, $\overline{DE}$, $\overline{CE}$

VERTICES: C, D, E

ANGLES: ∠CDE or ∠D
∠CED or ∠E
∠DCE or ∠C



The side opposite ∠C is $\overline{DE}$.
The angle opposite $\overline{CE}$ is ∠D.
∠E is opposite $\overline{CD}$.

## CLASSIFYING TRIANGLES BY THEIR ANGLES

ACUTE TRIANGLE - all 3 angles are acute

OBTUSE TRIANGLE - 1 angle is obtuse

RIGHT TRIANGLE - 1 angle is right (90 degrees)

EQUIANGULAR TRIANGLE - an acute triangle in which all angles are congruent

# CLASSIFYING TRIANGLES BY THEIR SIDES

SCALENE TRIANGLE - no 2 sides are congruent

ISOSCELES TRIANGLE - at least 2 sides are congruent

EQUILATERAL TRIANGLE - all 3 sides are congruent

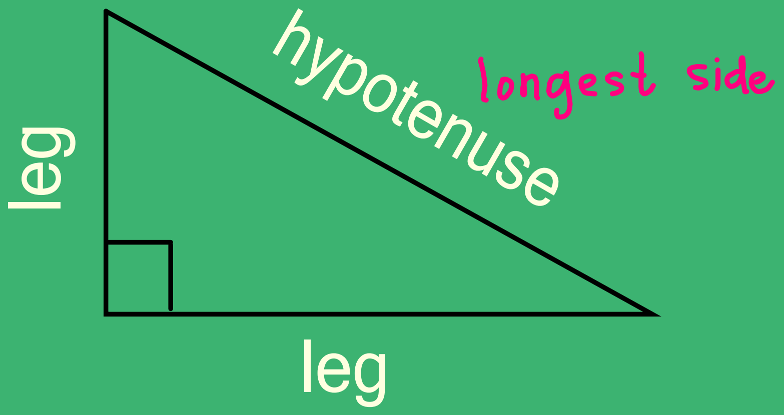
Example 1

The Alcoa Office Building shown at the left is located in San Francisco, California. Triangular bracings help to secure the building in the event of high winds or an earthquake. Classify ΔABC, ΔBCD, and ΔBCE.

| ΔABC | ΔBCD | ΔBCE |
|---|---|---|
| equiangular<br>acute<br>equilateral<br>isosceles | right<br>scalene | obtuse<br>isosceles |

SPECIAL PARTS OF A TRIANGLE

## RIGHT TRIANGLE

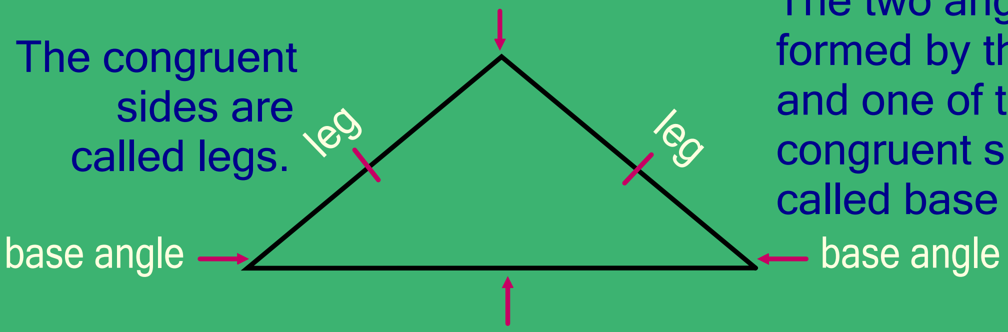


SPECIAL PARTS OF A TRIANGLE

## ISOSCELES TRIANGLE

The angle formed by
the congruent sides is
called the vertex angle

The congruent
sides are
called legs.



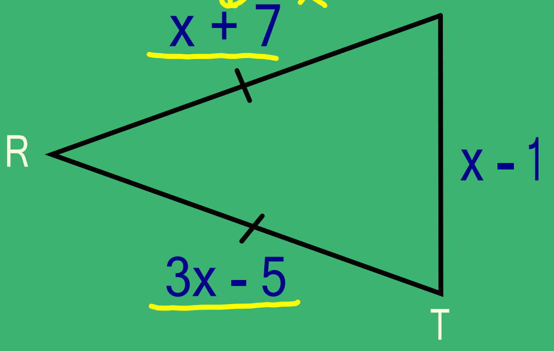
The two angles
formed by the base
and one of the
congruent sides are
called base angles.

The side opposite the vertex
angle is called the base

## Example 2

Triangle RST is isosceles,
∠ R is the vertex angle,
RS = x + 7, ST = x - 1,
and RT = 3x - 5.
Find x, RS, ST, and RT.

$x + 7 = 3x - 5$
$-x \qquad -x$
$7 = 2x - 5$
$+5 \qquad +5$
$12 = 2x$
$6 = x$



$\boxed{\begin{array}{l} x = 6 \\ RS = 13 \\ ST = 5 \\ RT = 13 \end{array}}$

$RS = x + 7$
$= 6 + 7$

$ST = x - 1$
$= 6 - 1$

$RT = 3x - 5$
$= 3(6) - 5$

## Example 3

Triangle PQR is an equilateral triangle.
One side measures 2x + 5 and another side
measures x + 35. Find the length of each side.

$2x + 5 = x + 35$
$-x \qquad -x$
$x + 5 = 35$
$-5 \qquad -5$
$x = 30$

$x + 35 = 30 + 35$

$\boxed{\text{all sides} = 65}$

## Example 4

Given Triangle STU with vertices S(2, 3), T(4, 3), and U(3, -2), use the distance formula to prove Triangle STU is isosceles.

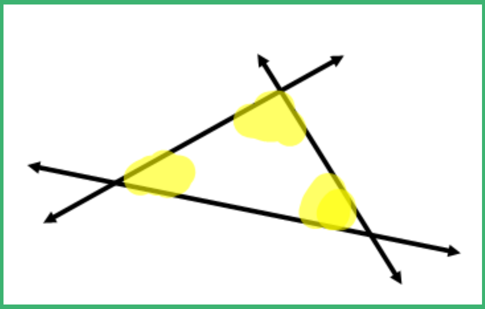
$$ST=\sqrt{(4-2)^2+(3-3)^2}$$
$$=\sqrt{(2)^2+(0)^2}$$
$$=\sqrt{4+0}=\sqrt{4}=2 \quad \longrightarrow ST=2$$

$$TU=\sqrt{(3-4)^2+(-2-3)^2} \quad \longrightarrow TU=\sqrt{26}$$
$$=\sqrt{(-1)^2+(-5)^2}$$
$$=\sqrt{1+25}=\sqrt{26}$$

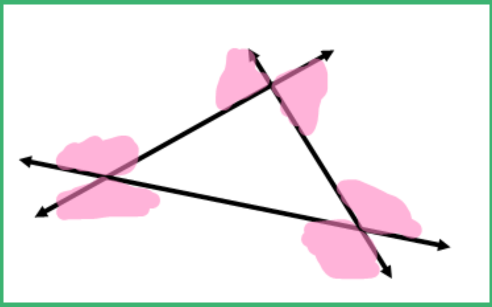
$$US=\sqrt{(3-2)^2+(-2-3)^2} \quad \longrightarrow US=\sqrt{26}$$
$$=\sqrt{(1)^2+(-5)^2}$$
$$=\sqrt{1+25}=\sqrt{26}$$

$\}$ isosceles b/c 2 sides are $\cong$

Interior Angles



Exterior Angles

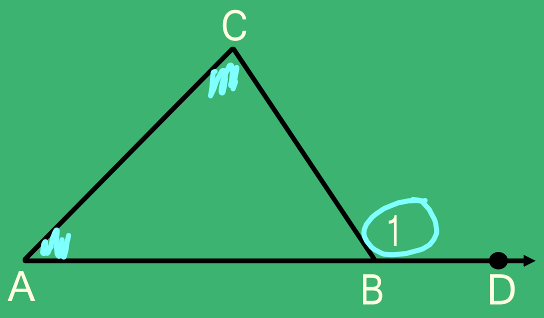


## Theorem 4.1: Angle Sum Theorem

The sum of the measures of the interior angles of a triangle is 180.

## Theorem 4.3: Exterior Angle Theorem

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.



$m\angle 1 = m\angle A + m\angle C$

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A <u>corollary</u> to a theorem is a statement that can be proved easily by using a theorem.

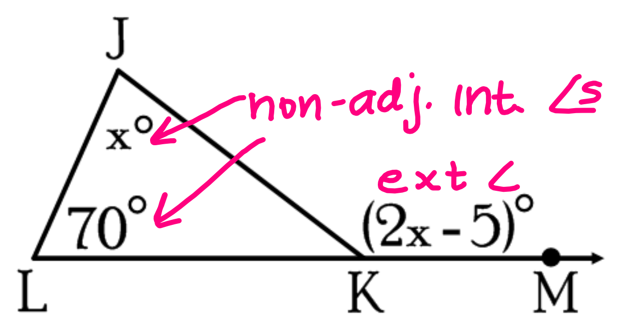
## Corollary to the Triangle Sum Theorem

The acute angles of a right triangle are complementary.

↓ one ∠ = 90°

## Example 5

Find the measure of ∠JKM.



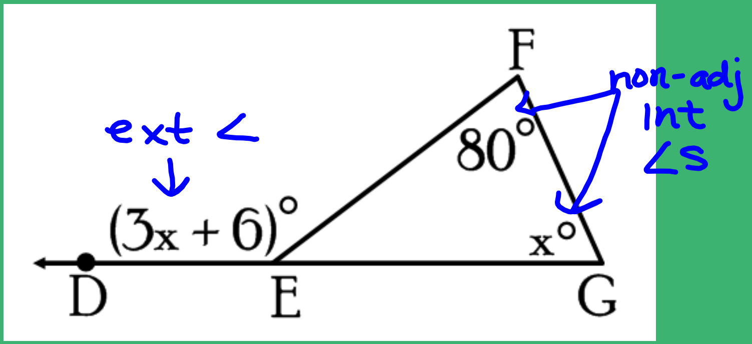
$$\cancel{x} + 70 = 2x - 5$$
$$\cancel{-x} \qquad -x$$
$$70 = x - 5$$
$$+5 \qquad +5$$
$$75 = x$$

$m\angle JKM = 2(75) - 5$

$$\boxed{m\angle JKM = 145^\circ}$$

## Example 6

Find the measure of ∠DEF.



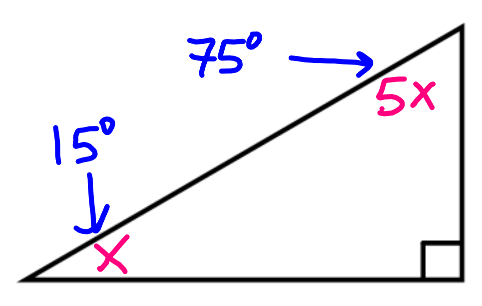
$$80 + x = 3x + 6$$
$$-x \qquad -x$$
$$80 = 2x + 6$$
$$-6 \qquad -6$$
$$\frac{74}{2} = \frac{2x}{2}$$
$$37 = x$$

$m\angle DEF = 3(37) + 6$

$$\boxed{m\angle DEF = 117^\circ}$$

## Example 7

The support for the skateboard ramp shown forms a right angle. The measure of one acute angle in the triangle is five times the measure of the other. Find the measure of each acute angle.



$5x + x + 90 = 180$

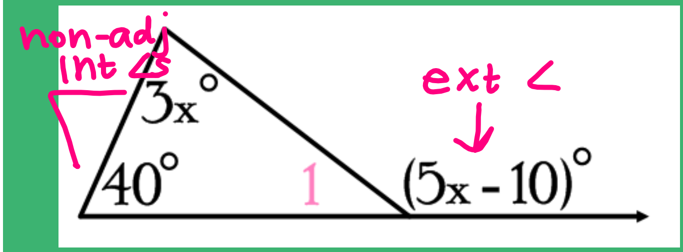
or

$5x + x = 90$

$x = 15$

## Example 8

Find the measure of ∠1.



$3x + 40 = 5x - -10$
$-3x \qquad -3x$

$40 = 2x - 10$
$+10 \qquad +10$

$\frac{50}{2} = \frac{2x}{2}$

$25 = x$

$40 + 3(\overset{75}{\cancel{25}}) + m\angle 1 = 180$

$115 + m\angle 1 = 180$

$\boxed{m\angle 1 = 65°}$

## Example 9

Find the measure of each interior angle of Trian
ABC, where m∠A = x°, m∠B = 2x°, and m∠C = 3x°

$$x + 2x + 3x = 180$$

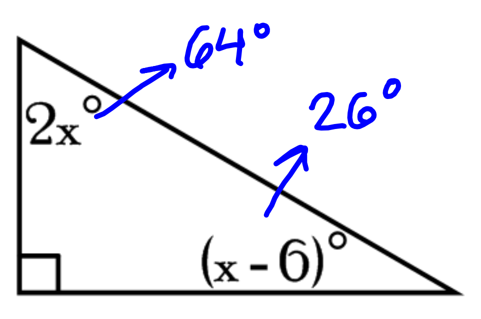
$$\frac{6x}{6} = \frac{180}{6}$$

$$x = 30$$

$$\boxed{\begin{aligned} m\angle A &= 30^\circ \\ m\angle B &= 60^\circ \\ m\angle C &= 90^\circ \end{aligned}}$$

## Example 10

Find the measure of the acute angles of the right triangle in the diagram shown.



$$2x + (x-6) + 90 = 180$$

$$\begin{array}{r} 3x + 84 = 180 \\ -84 \quad -84 \\ \hline \end{array}$$

$$\frac{3x}{3} = \frac{96}{3}$$

$$x = 32$$

## Example 11

Find the measure of each numbered angle in the figure below.

$m\angle 1 = \underline{65} + \underline{46}$

$m\angle 1 + m\angle 2 = 180$

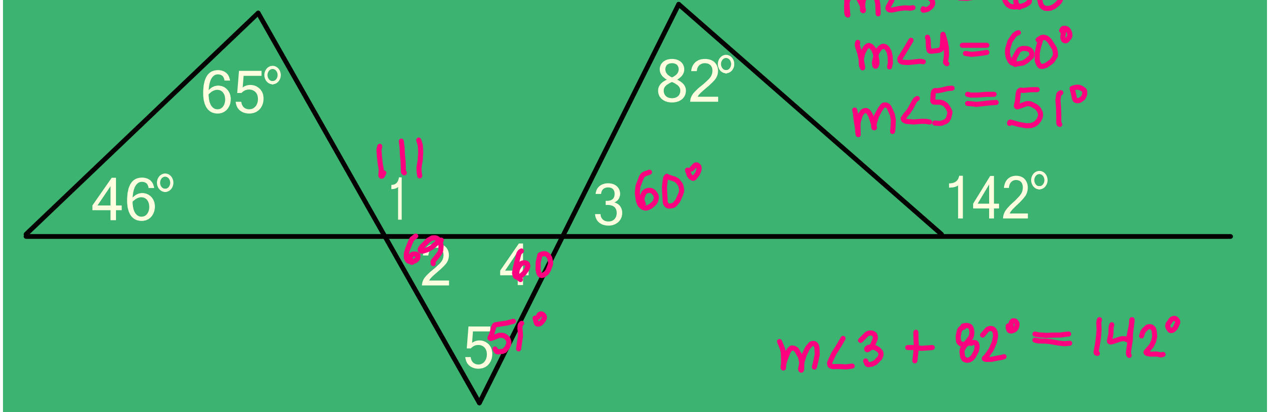
$m\angle 1 = 111°$

$m\angle 2 = 69°$

$m\angle 3 = 60°$

$m\angle 4 = 60°$

$m\angle 5 = 51°$



$m\angle 3 + 82° = 142°$

$m\angle 2 + m\angle 4 + m\angle 5 = 180$

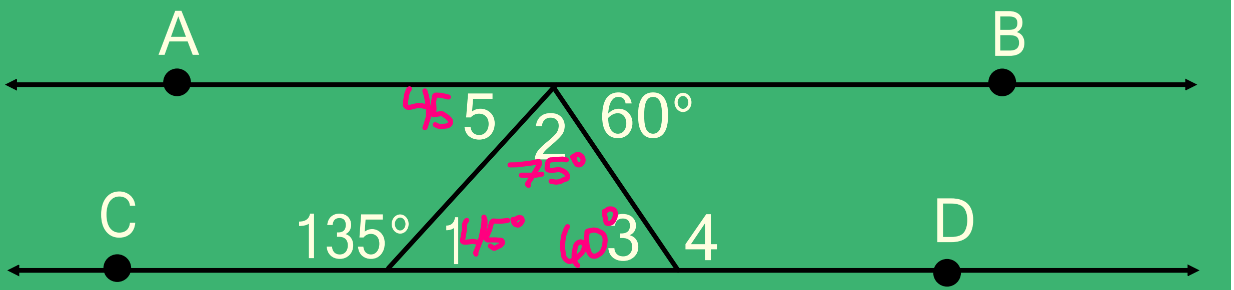
$129 + m\angle 5 = 180$

## Example 12

Find the measure of each numbered angle in the figure if $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.

$45 + 60 + m\angle 2 = 180$

$105 + m\angle 2 = 180$



$m\angle 1 + 135° = 180°$

$m\angle 5$ & $m\angle 1$ = same/alt. Int. ∠s

$m\angle 3$ & $60°$ = same

$m\angle 3$ & $m\angle 4$ = linear pair

$m\angle 1 = 45°$

$m\angle 2 = 75°$

$m\angle 3 = 60°$

$m\angle 4 = 120°$

$m\angle 5 = 45°$