

5.4 Part 2 Completing the Square

Vertex Form of a Quadratic Function

$$y = a(x - h)^2 + k$$

(h, k) are the coordinates of the vertex

Recall **H**orizontal

Inside

Opposite

Vertical

Outside

Same

1. Given $g(x) = x^2 - 10x + 22$,
 - a) Write the function in **vertex form**
 - b) Give the **coordinates of the vertex**
 - c) Write the **equation of the axis of symmetry**
 - d) Describe the **transformations**.

$$\begin{array}{l}
 \text{a) } g(x) = x^2 - 10x + 22 \\
 \underline{-22 \qquad \qquad \qquad -22} \\
 g(x) - 22 + 25 = x^2 - 10x + 25 \\
 \qquad \qquad \qquad \frac{1}{2}(-10) = -5 \\
 \qquad \qquad \qquad (-5)^2 = 25
 \end{array}$$

$$\begin{array}{l}
 g(x) + 3 = (x - 5)^2 \\
 \underline{-3 \qquad \qquad \qquad -3}
 \end{array}$$

$$g(x) = (x - 5)^2 - 3$$

b) vertex $(5, -3)$

c) a.o.s. $x = 5$

d) hor. translation
5 units right
vert. translation
3 units down

2. Given $y = x^2 + 12x - 37$,
- Write the function in vertex form
 - Give the coordinates of the vertex
 - Write the equation of the axis of symmetry
 - Describe the transformations.

$$\begin{aligned} \text{a) } y &= x^2 + 12x - 37 \\ &\quad \quad \quad +37 \quad \quad \quad +37 \\ \hline y + 37 + 36 &= x^2 + 12x + 36 \\ &\quad \quad \quad \frac{1}{2}(12) = 6 \\ &\quad \quad \quad (6)^2 = 36 \end{aligned}$$

$$\begin{aligned} y + 73 &= (x + 6)^2 \\ -73 \quad -73 \\ \hline \end{aligned}$$

$$y = (x + 6)^2 - 73$$

$$\text{b) vertex } (-6, -73)$$

$$\text{c) a.o.s. } x = -6$$

d) hor. translation
6 units left
vert. translation
73 units down

3. Given $f(x) = 2x^2 - 8x + 17$,
- Write the function in vertex form
 - Give the coordinates of the vertex
 - Write the equation of the axis of symmetry
 - Describe the transformations.

$$\begin{aligned} \text{a) } f(x) &= 2x^2 - 8x + 17 \\ f(x) - 17 &= 2x^2 - 8x \\ f(x) - 17 + 8 &= 2(x^2 - 4x + 4) \\ &\quad \quad \quad \frac{1}{2}(-4) = -2 \\ &\quad \quad \quad (-2)^2 = 4 \end{aligned}$$

$$f(x) - 9 = 2(x - 2)^2$$

$$f(x) = 2(x - 2)^2 + 9$$

$$\text{b) vertex } (2, 9)$$

$$\text{c) a.o.s. } x = 2$$

d) vert. stretch by a
factor of 2
hor. translation 2 units
right
vert. translation 9 units
up

4. Given $y = 3x^2 + 9x - 1$,
- Write the function in vertex form
 - Give the coordinates of the vertex
 - Write the equation of the axis of symmetry
 - Describe the transformations.

a) $y = 3x^2 + 9x - 1$
 $y + 1 = 3x^2 + 9x$
 $y + 1 + \frac{27}{4} = 3(x^2 + 3x + \frac{9}{4})$
 $\frac{1}{2}(3) = \frac{3}{2}$
 $(\frac{3}{2})^2 = \frac{9}{4}$
 $y + \frac{31}{4} = 3(x + \frac{3}{2})^2$
 $y = 3(x + \frac{3}{2})^2 - \frac{31}{4}$

b) vertex $(-\frac{3}{2}, -\frac{31}{4})$

c) a.o.s. $x = -\frac{3}{2}$

d) vert. stretch by a factor of 3

hor. translation $\frac{3}{2}$ units left

vert. translation $\frac{31}{4}$ units down

5. Given $y = \frac{1}{2}x^2 + 6x + 3$,
- Write the function in vertex form
 - Give the coordinates of the vertex
 - Write the equation of the axis of symmetry
 - Describe the transformations.

a) $y = \frac{1}{2}x^2 + 6x + 3$
 $y - 3 = \frac{1}{2}x^2 + 6x$
 $y - 3 + 18 = \frac{1}{2}(x^2 + 12x + 36)$
 $\frac{1}{2}(12) = 6$
 $(6)^2 = 36$
 $y + 15 = \frac{1}{2}(x + 6)^2$
 $y = \frac{1}{2}(x + 6)^2 - 15$

b) vertex $(-6, -15)$

c) a.o.s. $x = -6$

d) vert. compression by a factor of $\frac{1}{2}$

hor. translation 6 units left

vert. translation 15 units down

6. Given $y = -x^2 + 18x + 95$,
- Write the function in vertex form
 - Give the coordinates of the vertex
 - Write the equation of the axis of symmetry
 - Describe the transformations.

$$\begin{aligned}
 \text{a) } y &= -x^2 + 18x + 95 \\
 y - 95 &= -x^2 + 18x \\
 y - 95 - 81 &= -(x^2 - 18x + 81) \\
 &\quad \frac{1}{2}(-18) = -9 \\
 &\quad (-9)^2 = 81 \\
 y - 176 &= -(x - 9)^2 \\
 y &= -(x - 9)^2 + 176
 \end{aligned}$$

$$\text{b) vertex } (9, 176)$$

$$\text{c) a.o.s. } x = 9$$

- d) vert. reflection
over x-axis
hor. translation 9 units
right
vert. translation 176 units
up

7. Given $y = -\frac{1}{4}x^2 - 4x - 4$,
- Write the function in vertex form
 - Give the coordinates of the vertex
 - Write the equation of the axis of symmetry
 - Describe the transformations.