2.5 TRANSFORMATIONS OF FUNCTIONS

3 Types of Transformations

1. Translation: moving the graph up, down, left or right
2. Stretch or Compression: stretching or "squishing" the graph
3. Reflection: reflecting the graph across an axis

HIOVOS: A trick for remembering

Horizontal

Inside

Opposite

Vertical

Outside

Same

If you see...	Then it means...
Addition or Subtraction	**Translation** Horizontal: "inside" the function/OPPOSITE (left or right) Vertical: "outside" the function/SAME (up or down)
Multiplication (Integers & Fractions)	**Stretch or Compression** Horizontal: "inside" the function/OPPOSITE (reciprocal)Vertical: "outside" the function/SAME State the factor by which the graph is stretched (greater than 1) or compressed (greater than zero/less than 1)
Negative Sign	**Reflection** Horizontal: "inside" the function (over y-axis) Vertical: "outside" the function (over x-axis)

TRANSLATION

Original: $y = x^2$

Inside: $y = (x + 2)^2$ hor. translation 2 units left

$y = (x - 2)^2$ hor. translation 2 units right

Outside: $y = x^2 + 2$ vert. translation 2 units up

$y = x^2 - 2$ vert. translation 2 units down

STRETCH/COMPRESSION

Original: $y = x^2$

Inside: $y = (2x)^2$ hor. compression by a factor of $\frac{1}{2}$

$y = (\frac{1}{2}x)^2$ hor. stretch by a factor of 2

Outside: $y = 2x^2$ vert stretch by a factor of 2

$y = \frac{1}{2}x^2$ vert. compression by a factor of $\frac{1}{2}$

REFLECTION

Original: $y = x^2$

Inside: $y = (-x)^2$ hor. reflection over y-axis

Outside: $y = -x^2$ vert. reflection over x-axis

Describe the transformations of each function.

1. $y = 2(x+1)^2$

vert. stretch by a factor of 2
hor. translation 1 unit right

2. $y = -\sqrt{x+4} - 5$

vert. reflection over x-axis
hor. translation 4 units left
vert. translation 5 units down

3. $y = -4|x-3| + 2$

vert. reflection over x-axis
vert. stretch by a factor of 4
hor. translation 3 units right
vert. translation 2 units up

4. $y = 3[2x] - 6$

vert. stretch by a factor of 3
hor. compression by a factor of 1/2
vert. translation 6 units down

5. $y = -\frac{1}{2}x^3 + 3$

vert. reflection over x-axis
vert. compression by a factor of 1/2
vert. translation 3 units up

6. $y = 3\sqrt[3]{\frac{1}{2}x - 1} + 1$

$y = 3\sqrt[3]{\frac{1}{2}(x-2)} + 1$

$1 \div \frac{1}{2} \rightarrow 1 \cdot \frac{2}{1}$

vert. stretch by a factor of 3
hor. stretch by a factor of 2
hor. translation 2 units right
vert. translation 1 unit up

7. $y = \frac{-3}{x+4}$

$y = -3 \cdot \frac{1}{x+4}$

vert. reflection over x-axis
vert. stretch by a factor of 3
hor. translation 4 units left

8. $y = \frac{2}{x^2} - 3$

$y = 2 \cdot \frac{1}{x^2} - 3$

vert. stretch by a factor of 2
vert. translation 3 units down

Write an equation of the transformed function described below in each problem.

1. Quadratic: Vertical shift up 3 units, Horizontal shift to the left 2 units, Vertical compression by a factor of 1/2

$y = \frac{1}{2}(x+2)^2 + 3$

2. Cube Root: Vertical shift down 4 units, Reflection over the x-axis, Horizontal compression by a factor of 1/2

$y = -\sqrt[3]{2x} - 4$

3. Absolute Value: Vertical shift up 1 unit, Horizontal shift to the right 5 units, Vertical stretch by a factor of 2

$y = 2|x-5| + 1$

4. Greatest Integer Function: Horizontal shift to the left 4 units, Horizontal stretch by a factor of 4, Reflection over the x-axis, Vertical shift down 8 units

$y = -[\frac{1}{4}(x+4)] - 8$