

5.4 Part 1 Completing the Square

Reminder: Factor each below.

$$1. \quad \begin{array}{c} x^2 + 14x + 49 \\ \underbrace{(x)^2} \quad \underbrace{2(x)(7)} \quad \underbrace{(7)^2} \\ \downarrow \\ (x+7)^2 \end{array}$$

$$2. \quad \begin{array}{c} x^2 - 8x + 16 \\ \underbrace{(x)^2} \quad \underbrace{2(x)(4)} \quad \underbrace{(4)^2} \\ \downarrow \\ (x-4)^2 \end{array}$$

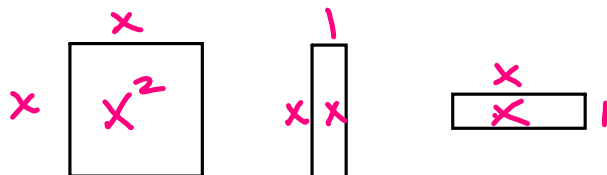
Find the value of c so you can factor as a **perfect square trinomial**.

$$3. \quad \begin{array}{c} x^2 - 20x + c \\ \underbrace{(x)^2} \quad \underbrace{2(x)(10)} \quad \underbrace{(10)^2} \end{array}$$

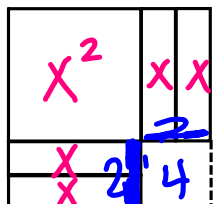
$$c = 100$$

$$4. \quad \begin{array}{c} x^2 + 16x + c \\ \underbrace{(x)^2} \quad \underbrace{2(x)(8)} \quad \underbrace{(8)^2} \end{array}$$

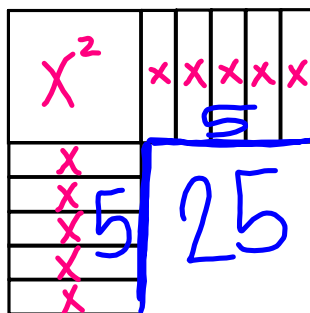
$$c = 64$$



$$x^2 + 4x + \underline{4}$$



$$x^2 + 10x + \underline{25}$$



COMPLETING THE SQUARE

- $ax^2 + bx + c$
- STEP 1:** Move the constant to the other side.
- STEP 2:** Find $\frac{1}{2}$ of b .
- STEP 3:** Square the result of Step 2.
- STEP 4:** Add the result of Step 3.
- STEP 5:** Factor and solve by square roots.

*** Important:** The coefficient of x^2 must be 1!

Solve by completing the square.

$$5. \quad x^2 + 6x - 16 = 0$$

$$\begin{array}{r} +16 \quad +16 \\ \hline x^2 + 6x + 9 = 16 + 9 \end{array}$$

perf. sq. trinomial

$$\frac{1}{2}(6) = 3$$

$$(3)^2 = 9$$

$$\sqrt{(x+3)^2} = \sqrt{25}$$

$$\begin{array}{r} x+3 = \pm 5 \\ -3 \quad -3 \\ \hline x = -3 \pm 5 \end{array}$$

$$\begin{array}{l} \swarrow \quad \searrow \\ x = -3 + 5 \quad x = -3 - 5 \\ \boxed{x = 2} \quad \boxed{x = -8} \end{array}$$

$$6. \quad x^2 - 40 = -18x$$

$$\begin{array}{r} +18x \quad +40 \quad +18x \quad +40 \\ \hline x^2 + 18x + 81 = 40 + 81 \end{array}$$

$$\frac{1}{2}(18) = 9$$

$$(9)^2 = 81$$

$$\sqrt{(x+9)^2} = \sqrt{121}$$

$$\begin{array}{r} x+9 = \pm 11 \\ -9 \quad -9 \\ \hline x = -9 \pm 11 \end{array}$$

$$\begin{array}{l} \swarrow \quad \searrow \\ x = -9 + 11 \quad x = -9 - 11 \\ \boxed{x = 2} \quad \boxed{x = -20} \end{array}$$

Solve by completing the square.

$$7. \quad x^2 = -12x - 4$$

$$\begin{array}{r} +12x \quad +12x \\ \hline x^2 + 12x + 36 = -4 + 36 \end{array}$$

$$\frac{1}{2}(12) = 6$$

$$(6)^2 = 36$$

$$2 \overline{) 32}$$

$$\sqrt{(x+6)^2} = \sqrt{32}$$

$$\begin{array}{r} x+6 = \pm 4\sqrt{2} \\ -6 \quad -6 \\ \hline \boxed{x = -6 \pm 4\sqrt{2}} \end{array}$$

$$\begin{array}{l} \swarrow \quad \searrow \\ x = -6 + 4\sqrt{2} \quad x = -6 - 4\sqrt{2} \end{array}$$

$$8. \quad 2x^2 + 8x - 16 = 0$$

$$\begin{array}{r} +16 \quad +16 \\ \hline 2x^2 + 8x = 16 \end{array}$$

$$2(x^2 + 4x + 4) = 16 + 8$$

$$\frac{1}{2}(4) = 2$$

$$(2)^2 = 4$$

$$2 \overline{) 12}$$

$$2 \overline{) 6}$$

$$\frac{2}{2} \cdot (x+2)^2 = \frac{24}{2}$$

$$\sqrt{(x+2)^2} = \sqrt{12}$$

$$\begin{array}{r} x+2 = \pm 2\sqrt{3} \\ -2 \quad -2 \\ \hline \boxed{x = -2 \pm 2\sqrt{3}} \end{array}$$

Solve by completing the square.

$$9. x^2 = 1 - 3x$$

$$\begin{array}{r} +3x \quad +3x \\ \hline x^2 + 3x + \frac{9}{4} = 1 + \frac{9}{4} \end{array}$$

$$\frac{1}{2}(3) = \frac{3}{2}$$

$$\left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$\sqrt{\left(x + \frac{3}{2}\right)^2} = \sqrt{\frac{13}{4}}$$

$$x + \frac{3}{2} = \pm \frac{\sqrt{13}}{2}$$

$$\begin{array}{r} -\frac{3}{2} \quad -\frac{3}{2} \\ \hline \boxed{x = -\frac{3}{2} \pm \frac{\sqrt{13}}{2}} \end{array}$$

$$10. 3x^2 - 5 = 6x$$

$$\begin{array}{r} -6x \quad +5 \\ \hline 3x^2 - 6x = 5 \end{array}$$

$$3(x^2 - 2x + 1) = 5 + 3$$

$$\frac{1}{2}(-2) = -1$$

$$(-1)^2 = 1$$

$$\frac{3}{3} \cdot (x-1)^2 = \frac{8}{3}$$

$$\sqrt{(x-1)^2} = \sqrt{\frac{8}{3} \cdot \frac{\sqrt{3}}{\sqrt{3}}}$$

$$x-1 = \pm \frac{\sqrt{24}}{3}$$

$$x-1 = \pm \frac{2\sqrt{6}}{3}$$

$$\begin{array}{r} +1 \quad +1 \\ \hline \boxed{x = 1 \pm \frac{2\sqrt{6}}{3}} \end{array}$$

$$\begin{array}{r} 2 \overline{)24} \\ 2 \overline{)12} \\ \hline 2 \overline{)6} \\ \hline 3 \end{array}$$

Solve by completing the square.

$$11. 2x^2 + 10x = 6$$

$$\begin{array}{r} 2(x^2 + 5x + \frac{25}{4}) = 6 + \frac{25}{2} \end{array}$$

$$\frac{1}{2}(5) = \frac{5}{2}$$

$$\left(\frac{5}{2}\right)^2 = \frac{25}{4}$$

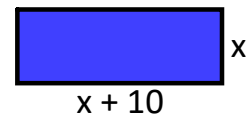
$$\frac{1}{2} \cdot 2 \left(x + \frac{5}{2}\right)^2 = \frac{37}{2} \cdot \frac{1}{2}$$

$$\sqrt{\left(x + \frac{5}{2}\right)^2} = \sqrt{\frac{37}{4}}$$

$$x + \frac{5}{2} = \pm \frac{\sqrt{37}}{2}$$

$$\begin{array}{r} -\frac{5}{2} \quad -\frac{5}{2} \\ \hline \boxed{x = -\frac{5}{2} \pm \frac{\sqrt{37}}{2}} \end{array}$$

12. The area of the rectangle is 50 square units. What is the value of x?



$$A = lw$$

$$x(x+10) = 50$$

$$x^2 + 10x + 25 = 50 + 25$$

$$\frac{1}{2}(10) = 5$$

$$(5)^2 = 25$$

$$\sqrt{(x+5)^2} = \sqrt{75}$$

$$x+5 = \pm 5\sqrt{3}$$

$$\begin{array}{r} -5 \quad -5 \\ \hline \end{array}$$

$$x = -5 \pm 5\sqrt{3}$$

$$\boxed{x = -5 + 5\sqrt{3}} \quad -5 - 5\sqrt{3} = x$$

$$\begin{array}{r} 5 \overline{)75} \\ 3 \overline{)15} \\ \hline 5 \end{array}$$