Solving

$ax^2 + bx + c = 0$

by

FACTORING

Quadratic Equations In Standard Form

$ax^2 + bx + c = 0$

NOTE:

- The solutions of a quadratic equation are called the roots of the equation.

AND

- Since the function's value (y) is zero when $ax^2 + bx + c = 0$, the solutions are also called zeros of the function $f(x) = ax^2 + bx + c$.

To solve $ax^2 + bx + c = 0$:

Use the "zero product property".

If A • B = 0, then A = 0 or B = 0

a. $3x - 6 = x^2 - 10$

$3x - 6 = x^2 - 10$

$-3x + 6 \quad +6 - 3x$

$0 = x^2 - 3x - 4$

sum -3 product -4

$\frac{-4}{1} \quad \frac{1}{1}$

1. Set = to 0 (may need to move terms).
2. Factor.
3. Set each factor = to 0.
4. Solve for the variable.

$0 = (x-4)(x+1)$

$x - 4 = 0 \quad x + 1 = 0$

$+4 \quad +4 \quad -1 \quad -1$

$x = 4 \quad x = -1$

b. Find the zeros of $f(x) = 3x^2 + 10x - 8$.

$0 = 3x^2 + 10x - 8$

sum 10 product -24

$4 \quad \frac{12}{3} \quad \frac{-2}{3}$

$0 = (x+4)(3x-2)$

$x + 4 = 0 \quad 3x - 2 = 0$

$x = -4 \quad x = \frac{2}{3}$

Find x

c. What are the roots of the equation $x^2 - 5x - 36 = 0$?

sum −5 product −36

$-9 \quad 4$

$(x-9)(x+4)=0$

$x-9=0$ $\quad x+4=0$

$\boxed{x=9}$ $\quad \boxed{x=-4}$

d. $3x^2 + 4x = 4$

$-4 \quad -4$

$3x^2+4x-4=0$

s 4 p −12

$2 \quad 6 \quad -2$

$\frac{2}{1} \quad \frac{6}{3} \quad \frac{-2}{3}$

$(x+2)(3x-2)=0$

$x+2=0$ $\quad 3x-2=0$

$\boxed{x=-2}$ $\quad \boxed{x=\frac{2}{3}}$

e. $16x^2 = 49$

$-49 \quad -49$

$16x^2-49=0$

$(4x)^2 \quad (7)^2$

$(4x+7)(4x-7)=0$

$4x+7=0$ $\quad 4x-7=0$

$\boxed{x=-\frac{7}{4}}$ $\quad \boxed{x=\frac{7}{4}}$

f. $3x^2 + 24x + 45 = 0$

$3(x^2 + 8x + 15) = 0$

s 8 p 15

3 5

$3(x+3)(x+5) = 0$

$3 \neq 0$ $x+3=0$ $x+5=0$

$x = -3$ $x = -5$

g. $10x^2 = 9x$

$-9x \quad -9x$

$10x^2 - 9x = 0$

$x(10x - 9) = 0$

$x = 0$ $10x - 9 = 0$

$x = \frac{9}{10}$

Practice

Solve by factoring.

1. $4x^2 = 24x$

2. $16x^2 - 361 = 0$

$(4x)^2 \quad (19)^2$

$(4x-19)(4x+19) = 0$

$4x - 19 = 0$ $4x + 19 = 0$

$x = \frac{19}{4}$ $x = -\frac{19}{4}$

3. $20x = 25x^2 + 4$

$0 = 25x^2 - 20x + 4$

$(5x)^2 \quad 2(5x)(2) \quad (2)^2$

$0 = (5x-2)^2$

$5x - 2 = 0$

$x = \frac{2}{5}$

4. $2x^2 + 7x - 15 = 0$

Answers

Solve by factoring.

1. $4x^2 = 24x$

   $x = 0, 6$

2. $16x^2 - 361 = 0$

   $x = \pm 19/4$

3. $20x = 25x^2 + 4$

   $x = 2/5$

4. $2x^2 + 7x - 15 = 0$

   $x = -5, 3/2$

Word Problems AGAIN!!

Doubling Area

$A = lw$

EXAMPLE 4 Use a quadratic equation as a model

NATURE PRESERVE A town has a nature preserve with a rectangular field that measures 600 meters by 400 meters. The town wants to double the area of the field by adding land as shown. Find the new dimensions of the field.

Solution

600 + 200 → 800

400 + 200 → 600

New area (square meters)	=	New length (meters)	·	New width (meters)

$480000 = (600+x)(400+x)$

$480000 = 240000 + 600x + 400x + x^2$

$-480000 \quad -480000$

$0 = x^2 + 1000x - 240000$

sum 1000 product −240000

$1200 \quad -200$

$0 = (x + 1200)(x - 200)$

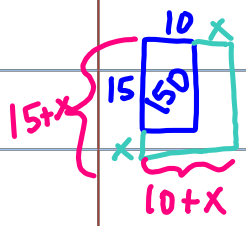
$x + 1200 = 0 \quad x - 200 = 0$

$x = -1200 \quad x = 200$

800m by 600m

Extra Example

You have a rectangular vegetable garden in your backyard that measures 15 feet by 10 feet. You want to double the area of the garden by adding the same distance x to the length and width of the garden. Find the value of x and the new dimensions of the garden.



$A = lw$

$300 = (15+x)(10+x)$

$300 = 150 + 15x + 10x + x^2$

$-300 \quad -300$

$0 = x^2 + 25x - 150$

sum 25 product -150

$30 \quad -5$

$0 = (x+30)(x-5)$

$x+30=0 \qquad x-5=0$

$x=-30 \qquad x=5$

$x = 5$

20 ft by 15 ft

Extra Example

A rectangular performing platform in a park measures 10 feet by 12 feet. You want to triple the platform's area by adding the same distance x to the length and the width. Find the new length and the width.