

## 5.3 FACTORING QUADRATICS

There are **several methods** available for solving a quadratic equation:

1. By Square Roots
2. By Factoring
3. By Completing the Square
4. By the Quadratic Formula
5. By Graphing

The method depends on the form of the equation.

### FACTORING QUADRATIC TRINOMIALS

Example  $5x^2 + 17x + 14$

↑  
leading coefficient

product

sum 17	product 70
1 · 70	-1 · -70
2 · 35	-2 · -35
<b>7 · 10</b>	<b>-7 · -10</b>

1. The expression must be in ascending or descending order.

2. Make a sum/product chart.

$$\frac{7}{5} \quad \frac{10}{5} \quad \frac{2}{1}$$

3. Divide each number by the leading coefficient.

4. **Reduce each fraction if possible.**

5. Denominator = constant or coefficient of first term  
 Numerator = constant or coefficient of last term

$$(5x + 7)(x + 2)$$

Examples:

a.  $x^2 + 6x + 8$

sum 6 product 8

$$\frac{4}{1} \quad \frac{2}{1}$$

$$(x+4)(x+2)$$

b.  $3x^2 - 11x + 6$

s -11 p 18

$$\frac{-3}{1} \quad \frac{-9}{3} \quad \frac{-2}{3}$$

$$(x-3)(3x-2)$$

Examples:

c.  $x^2 + 7x - 18$

s 7 p -18

$$\frac{-2}{1} \quad \frac{9}{1}$$

$$(x-2)(x+9)$$

d.  $3x^2 + 10x - 8$

s 10 p -24

$$\frac{4}{1} \quad \frac{12}{3} \quad \frac{-2}{3}$$

$$(x+4)(3x-2)$$

*Practice*

Factor each trinomial.

1)  $x^2 - 16x + 39$

2)  $x^2 + 2x - 35$

3)  $x^2 + 22x + 121$

4)  $x^2 - 2x - 63$

5)  $14x^2 - 11x + 2$

6)  $12x^2 + 16x - 3$

7)  $2x^2 + 13x + 6$

8)  $9x^2 - 9x - 28$

*Answers*

Factor each trinomial.

1)  $x^2 - 16x + 39$

$(x - 3)(x - 13)$

2)  $x^2 + 2x - 35$

$(x + 7)(x - 5)$

3)  $x^2 + 22x + 121$

$(x + 11)(x + 11)$

4)  $x^2 - 2x - 63$

$(x + 7)(x - 9)$

5)  $14x^2 - 11x + 2$

$(7x - 2)(2x - 1)$

6)  $12x^2 + 16x - 3$

$(2x + 3)(6x - 1)$

7)  $2x^2 + 13x + 6$

$(2x + 1)(x + 6)$

8)  $9x^2 - 9x - 28$

$(3x + 4)(3x - 7)$

## Special Factoring Patterns

### 1. FACTORING DIFFERENCE OF SQUARES

$$x^2 - 4 = (x - 2)(x + 2)$$

$$4x^2 - 9 = (2x - 3)(2x + 3)$$

$$x^2 - 49 = (x - 7)(x + 7)$$

$$64x^2 - 25 = (8x - 5)(8x + 5)$$

$$a^2 - b^2 = (a - b)(a + b)$$

What is the pattern?

## Special Factoring Patterns

### 2. PERFECT SQUARE TRINOMIALS

$$x^2 + 14x + 49 = (x + 7)^2$$

$(x)^2 \quad 2(x)(7) \quad (7)^2$

$$x^2 - 8x + 16 = (x - 4)^2$$

$$4x^2 - 20x + 25 = (2x - 5)^2$$

$$9x^2 + 12x + 4 = (3x + 2)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

$$a^2 + 2ab + b^2 = (a + b)^2$$

What is the pattern?

## Practice

Factor completely.

$$1. \quad 4x^2 - 121$$

$(2x)^2 \quad (11)^2$

$$(2x - 11)(2x + 11)$$

$$2. \quad 9x^2 - 24x + 16$$

$(3x)^2 \quad 2(3x)(4) \quad (4)^2$

$$(3x - 4)^2$$

$$3. \quad 225 - x^2$$

$(15)^2 \quad (x)^2$

$$(15 - x)(15 + x)$$

$$4. \quad x^2 + 10x + 25$$

$(x)^2 \quad 2(x)(5) \quad (5)^2$

$$(x + 5)^2$$

$$5. \quad 10x^2 - 13x - 3$$

$$(2x - 3)(5x + 1)$$

$s = -13 \quad p = -30$

$$\frac{-3 - 15}{2} \quad \frac{2}{10} \quad \frac{1}{5}$$

## Answers

Factor completely.

$$1. \quad 4x^2 - 121$$

$$(2x - 11)(2x + 11)$$

$$2. \quad 9x^2 - 24x + 16$$

$$(3x - 4)^2$$

$$3. \quad 225 - x^2$$

$$(15 - x)(15 + x)$$

$$4. \quad x^2 + 10x + 25$$

$$(x + 5)^2$$

$$5. \quad 10x^2 - 13x - 3$$

$$(2x - 3)(5x + 1)$$

When factoring,  
ALWAYS look for the GCF  
 first!

**Greatest Common Factor**

the largest factor that divides **ALL** of the terms

a.  $12x^2 - 3$

$$3(4x^2 - 1)$$

$(2x)^2 \quad (1)^2$

$$3(2x-1)(2x+1)$$

b.  $7v^2 - 42v$

$$7v(v-6)$$

FACTOR COMPLETELY

c.  $5x^2 - 45$

$$5(x^2 - 9)$$

$(x)^2 \quad (3)^2$

$$5(x-3)(x+3)$$

d.  $15x^2 + 6x$

$$3x(5x+2)$$

e.  $3x^2 - 9x + 6$

$$3(x^2 - 3x + 2)$$

$$\begin{array}{r} s-3p2 \\ -2-1 \\ \hline 1 \quad 1 \end{array}$$

$$3(x-2)(x-1)$$

f.  $36x - 48x^2 + 24x^3$

$$12x(3 - 4x + 2x^2)$$

$$\begin{array}{r} s=4 \quad p6 \\ \hline 1 \cdot 6 \quad -1 \cdot -6 \\ 2 \cdot 3 \quad -2 \cdot -3 \end{array}$$

*Practice*

Factor completely.

1.  $12x^2 - 3$

2.  $45x^2 + 10x$

$$3(4x^2 - 1)$$

$$(2x)^2 \quad (1)^2$$

$$3(2x - 1)(2x + 1)$$

3.  $8x^2 - 24x + 18$

4.  $x^2 + 5x + 4$

$$2(4x^2 - 12x + 9)$$

$$\frac{5-12p}{2} \quad \frac{36}{4}$$

$$\frac{3-6}{2} \quad \frac{-6-3}{4} \quad \frac{3}{2}$$

$$2(2x - 3)(2x - 3) \quad \text{or} \quad 2(2x - 3)^2$$

5.  $6x^2 + 13x - 5$

*Answers*

Factor completely.

1.  $12x^2 - 3$

2.  $45x^2 + 10x$

$$3(2x - 1)(2x + 1)$$

$$5x(9x + 2)$$

3.  $8x^2 - 24x + 18$

4.  $x^2 + 5x + 4$

$$2(2x - 3)^2$$

$$(x + 1)(x + 4)$$

5.  $6x^2 + 13x - 5$

$$(2x + 5)(3x - 1)$$

# FACTORIZING FOUR TERMS

When factoring four terms, use the  
grouping method.

a.  $x^2 - 12x + 3x - 36$     b.  $(ra + rb) + (sa + sb)$   
 $r(\cancel{a+b}) + s(\cancel{a+b})$   
 $(a+b)(r+s)$

FACTOR USING THE GROUPING METHOD.

c.  $(y^2 - 12y) + (4y + 48)$     d.  $(k^2 + 3k) + (8k - 24)$   
 $y(\cancel{y-12}) - 4(\cancel{y-12})$      $k(\cancel{k+3}) - 8(\cancel{k+3})$   
 $(y-12)(y-4)$      $(k+3)(k-8)$



*Practice*

Factor completely.

1.  $2x^2y - x + 6xy - 3$

2.  $(6cd^2 - 8cd)(-9d + 12)$

$2cd(\cancel{3d-4}) - 3(\cancel{3d-4})$

$(3d-4)(2cd-3)$

3.  $2xz - 6xy + 2yz - 6y^2$

$2[(xz - 3xy) + (yz - 3y^2)]$

$2[x(\cancel{z-3y}) + y(\cancel{z-3y})]$

$2(z-3y)(x+y)$

*Answers*

Factor completely.

1.  $2x^2y - x + 6xy - 3$

$(2xy - 1)(x + 3)$

2.  $6cd^2 - 8cd - 9d + 12$

$(2cd - 3)(3d - 4)$

3.  $2xz - 6xy + 2yz - 6y^2$

$2(x + y)(z - 3y)$