3.4 SLOPES OF LINES

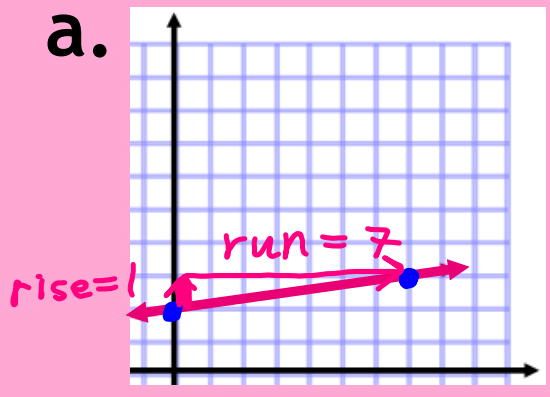
The slope of a line is the ratio of its vertical rise to the horizontal run.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$

Example 1

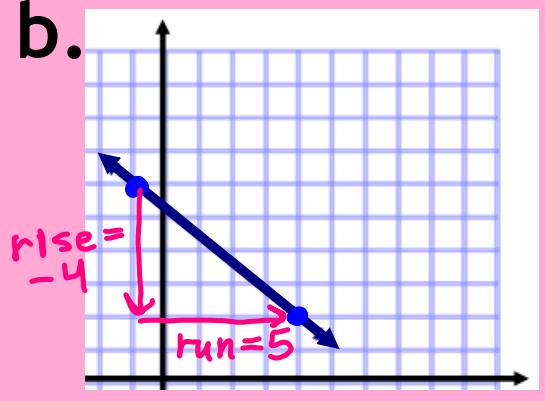
Find the slope of each line.

a.



$m = \frac{1}{7}$

b.

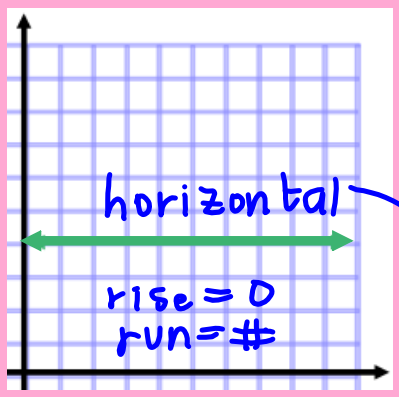


$m = -\frac{4}{5}$

Example 1

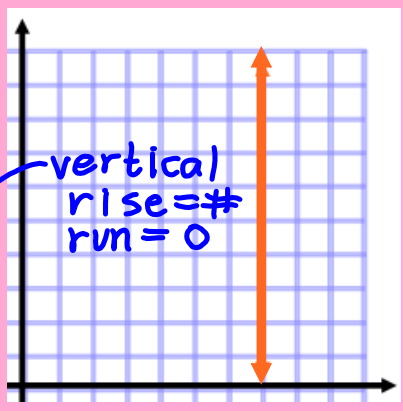
Find the slope of each line.

c.



$m = 0$

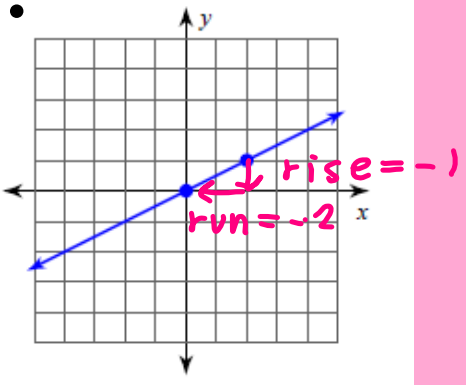
d.



undefined slope
no slope
(can't ÷ by 0)

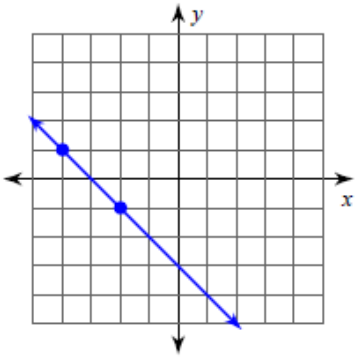
Find the slope of each line.

e.



$m = \frac{1}{2}$

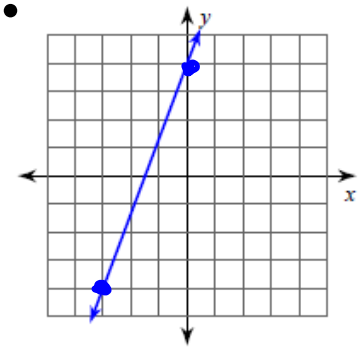
f.



$m = \frac{-2}{2} = -1$

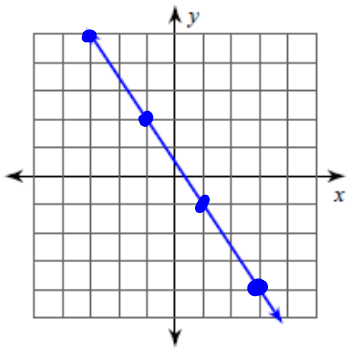
Find the slope of each line.

g.



$m = \frac{8}{3}$

h.



$m = -\frac{3}{2}$

Postulate 17 Slopes of Parallel Lines

In a coordinate plane, two nonvertical lines are parallel if and only if they have the same slope.

Any two vertical lines are parallel.

Postulate 18 Slopes of Perpendicular Lines

Two nonvertical lines are perpendicular if and only if the product of their slopes is -1.

Horizontal lines are perpendicular to vertical lines.

opposite reciprocal slopes

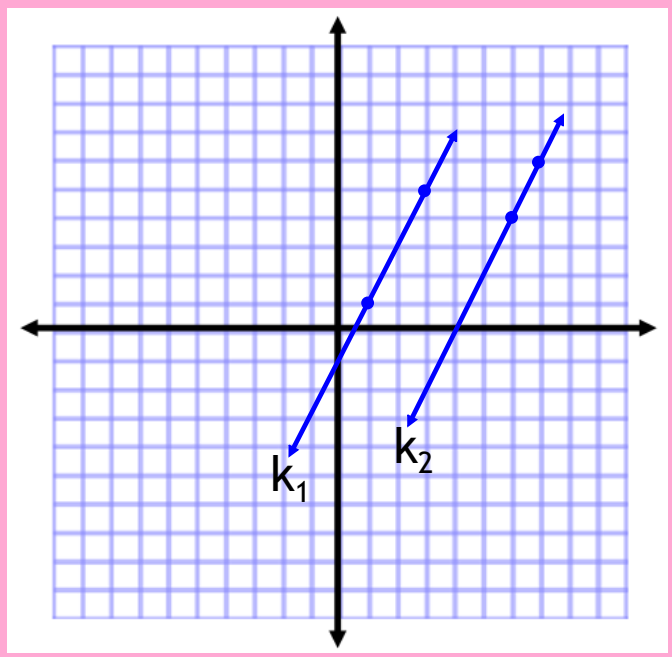
Example 2

Find the slope of each line. Is $k_1 \parallel k_2$?

$k_1 \rightarrow m = \frac{4}{2} = 2$

$k_2 \rightarrow m = \frac{2}{1} = 2$

$k_1 \parallel k_2$ b/c same slope



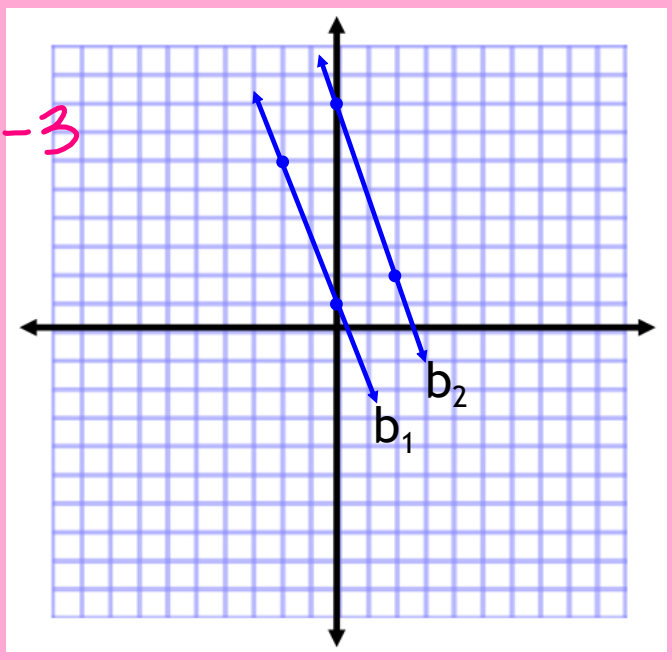
Example 3

Find the slope of each line. Is $b_1 \parallel b_2$?

$b_1 \rightarrow m = \frac{-5}{2}$

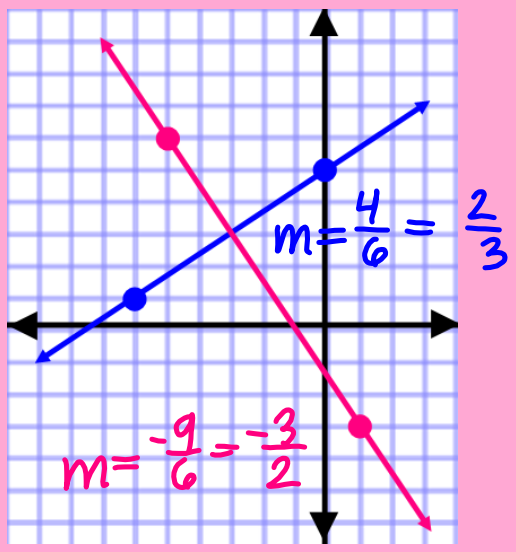
$b_2 \rightarrow m = -\frac{6}{2} = -3$

b_1 not $\parallel$ b_2 b/c not same slope



Example 4

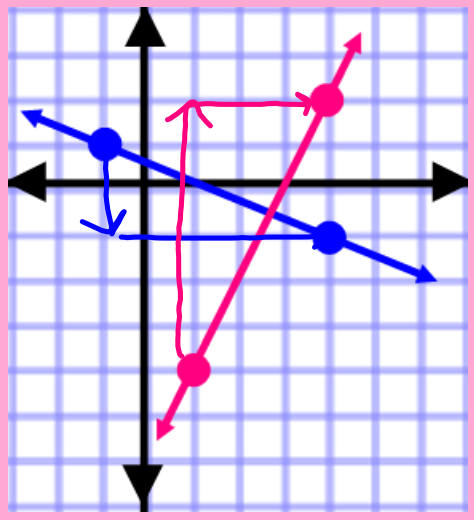
Find the slope of each line. Determine if the lines are perpendicular.



perpendicular b/c slopes are opposite reciprocals

Example 5

Find the slope of each line. Determine if the lines are perpendicular.



m of blue $= \frac{-2}{5}$

m of pink $= \frac{6}{3} = 2$

not $\perp$ b/c slopes aren't opp. rec.

Example 6

Determine whether lines p_1 and p_2 are perpendicular.

$y = mx + b$

line p_1: $y = 3x + 5$ $m = 3$

line p_2: $y = \frac{1}{3}x + 5$ $m = \frac{1}{3}$

p_1 & p_2 are not $\perp$ b/c slopes aren't opp. rec.

Example 7

Determine whether lines p_1 and p_2 are perpendicular.

$p_1 \perp p_2$

line p_1: $9x = 4 + 7y$

line p_2: $7x + 9y = -5$

$9x = 4 + 7y$

$-4 \quad -4$

$\frac{9x}{7} - \frac{4}{7} = \frac{7y}{7}$

$\frac{9}{7}x - \frac{4}{7} = y$

$m = \frac{9}{7}$

$7x + 9y = -5$

$-7x \quad -7x$

$\frac{9y}{9} = \frac{-7x}{9} - \frac{5}{9}$

$y = -\frac{7}{9}x - \frac{5}{9}$

$m = -\frac{7}{9}$

Example 8

Given A(-3,-2), B(9,1), C(3,6), and D(5,-2), determine if $\overleftrightarrow{AB}$ is *parallel* or *perpendicular* to $\overleftrightarrow{CD}$.

$\overleftrightarrow{AB}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{1 - -2}{9 - -3}$

$m = \frac{3}{12}$

$m = \frac{1}{4}$

$\overleftrightarrow{CD}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{-2 - 6}{5 - 3}$

$m = \frac{-8}{2}$

$m = \frac{-4}{1}$

$\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$

Example 9

Given P(-2,2), Q(2,1), R(1,-1), and S(5,-2), determine if $\overleftrightarrow{PQ}$ is *parallel* or *perpendicular* to $\overleftrightarrow{RS}$.

$\overleftrightarrow{PQ}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{1 - 2}{2 - -2}$

$m = \frac{-1}{4}$

$\overleftrightarrow{RS}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{-2 - -1}{5 - 1}$

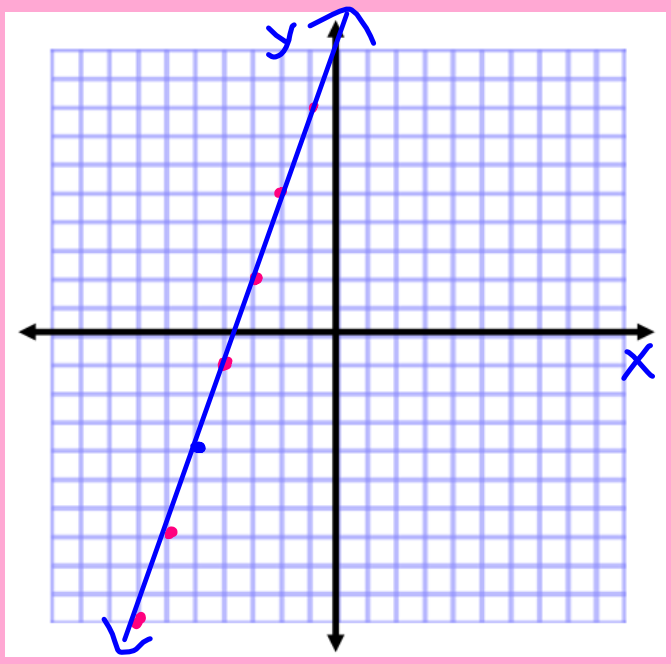
$m = \frac{-1}{4}$

$\overleftrightarrow{PQ} \parallel \overleftrightarrow{RS}$

Example 10

Graph the line through the point (-5,-4) with a slope of 3.

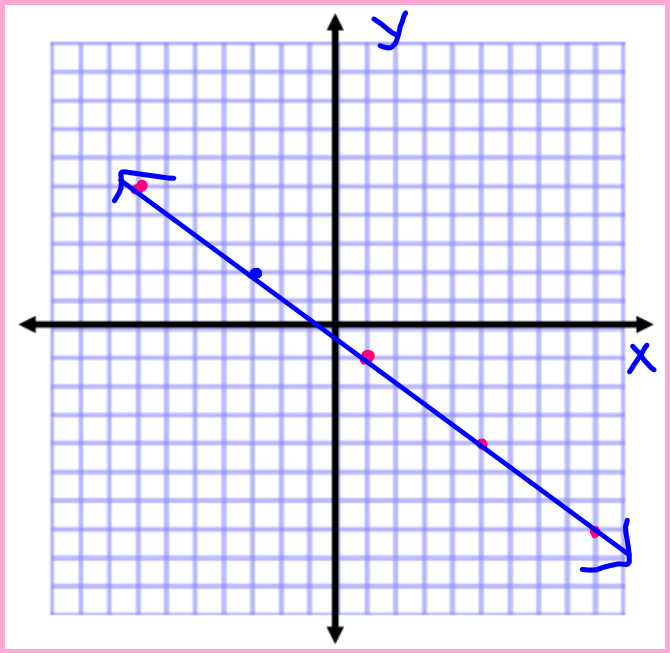
$m = \frac{3}{1}$ $\frac{\text{rise}}{\text{run}}$



Example 11

Graph the line through the point (-3,2) with a slope of $-\frac{3}{4}$.

$m = \frac{-3}{4}$ or $m = \frac{3}{-4}$



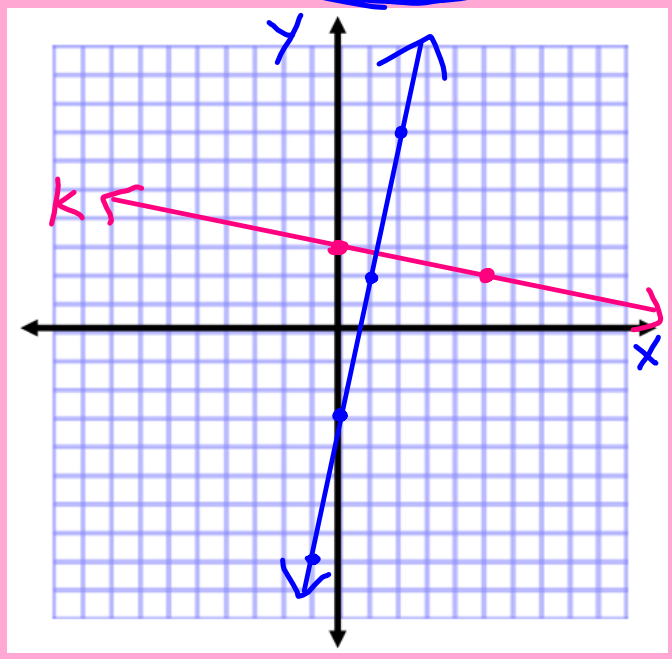
Example 12

$m = \frac{-1}{5}$

Line *k* passes through (0,3) and (5,2). Graph the line perpendicular to *k* that passes through (1,2).

opp. rec. slope

$m_{\perp} = \frac{5}{1}$ or 5



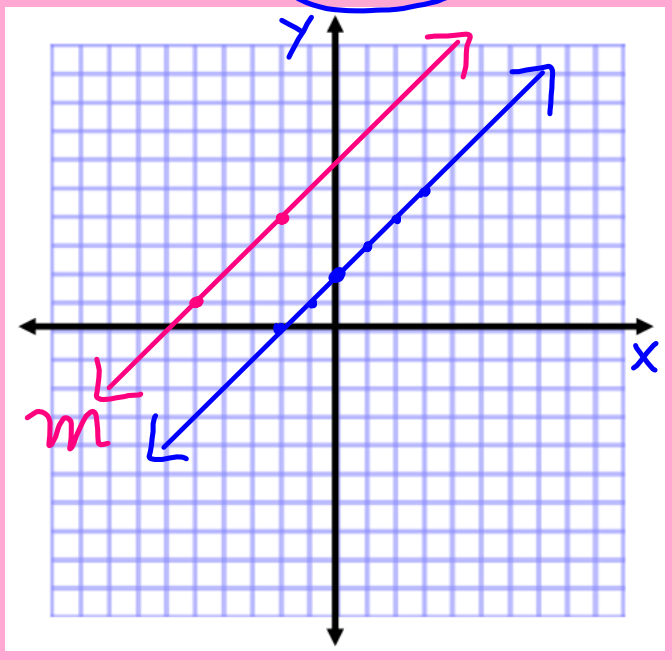
Example 13

$m = 1$

Line *m* passes through (-2,4) and (-5,1). Graph the line parallel to *m* that passes through (0,2).

Same slope

$m_{\parallel} = 1$



Example 14

Find the value of x so the line that passes through (x,5) and (6,-1) is perpendicular to the line that passes through (2,3) and (-3,-7).

opp. rec. slopes

(x_1, y_1) (x_2, y_2)

$$m = \frac{-7-3}{-3-2}$$

$$m = \frac{-10}{-5}$$

$$m = 2$$

$$m_\perp = -\frac{1}{2}$$

$$\frac{-1-5}{6-x} = \frac{-1}{2}$$

$$\frac{-6}{6-x} = \frac{-1}{2}$$

$$-6 \cdot 2 = -1 \cdot (6-x)$$

$$-12 = -6 + x$$

$$+6 \quad +6$$

$$-6 = x$$

Example 15

Find the value of x so the line that passes through (x,2) and (3,5) is perpendicular to the line that passes through (0,1) and (2,7).

(x_1, y_1) (x_2, y_2) (x_1, y_1) (x_2, y_2)

$$m = \frac{7-1}{2-0}$$

$$m = \frac{6}{2}$$

$$m = 3$$

$$m_\perp = -\frac{1}{3}$$

$$\frac{5-2}{3-x} = \frac{-1}{3}$$

$$\frac{3}{3-x} = \frac{-1}{3}$$

$$3 \cdot 3 = -1(3-x)$$

$$9 = -3 + x$$

$$+3 \quad +3$$

$$12 = x$$