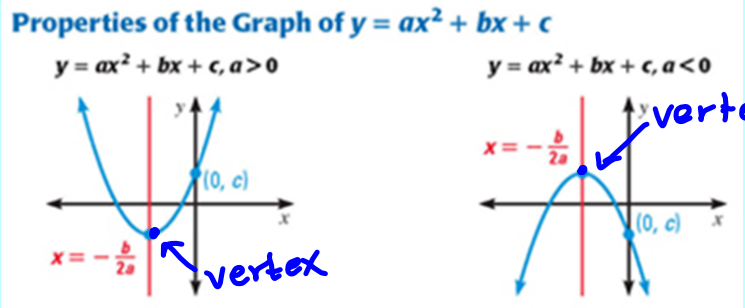
5.1 Graphing Quadratic Functions in Standard Form

Standard (Quadratic) Form: $y = ax^2 + bx + c$

A. Identify characteristics



- The graph opens up if $a > 0$ and down if $a < 0$.
- The graph is narrower than the graph of $y = x^2$ if $|a| > 1$ and wider if $|a| < 1$.
- The axis of symmetry is $x = -\frac{b}{2a}$ and the vertex has x-coordinate $-\frac{b}{2a}$.

Determine: a) whether the graph opens up or down
b) the axis of symmetry
c) the vertex

$y = ax^2 + bx + c$

Example: $f(x) = -x^2 - 2x + 1$

$a = -1 \quad b = -2$

a) graph opens down b/c a is negative

b) $x = \frac{-b}{2a} = \frac{2}{2(-1)} = \frac{2}{-2}$ → $x = -1$ a.o.s.

c) $y = -(-1)^2 - 2(-1) + 1$
$y = -1 + 2 + 1$
$y = 2$

Vertex $(-1, 2)$

Determine: a) whether the graph opens up or down
b) the axis of symmetry
c) the vertex

$a = \frac{2}{3}$ $b = -3$

Example: $f(x) = \frac{2}{3}x^2 - 3x + 6$

a) opens up b/c a is positive

b) $x = \frac{-b}{2a} = \frac{3}{2(\frac{2}{3})} = \frac{9}{4}$

a.o.s. $x = \frac{9}{4}$

c) $y = \frac{2}{3}(\frac{9}{4})^2 - 3(\frac{9}{4}) + 6$

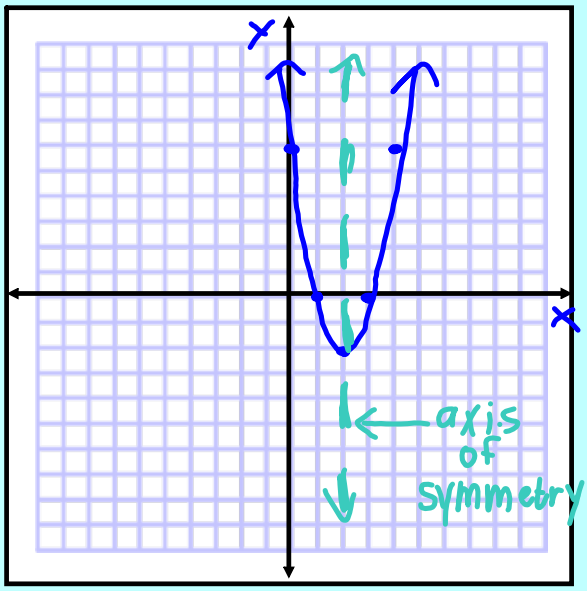
$y = \frac{21}{8}$

vertex $(\frac{9}{4}, \frac{21}{8})$

B. Graph Standard Form $a = 2$ $b = -8$

Example: $f(x) = 2x^2 - 8x + 6$ $x = \frac{-b}{2a} = \frac{8}{2(2)} = \frac{8}{4} = 2$

a.o.s. $x = 2$



1. Determine if the graph opens up or down.
2. Find the axis of symmetry.
3. Find the vertex & plot.
4. Make a table of values (find two points on either side of the vertex) & plot.
5. Connect the points with a smooth curve.

x		y
0	$2(0)^2 - 8(0) + 6$	6
1	$2(1)^2 - 8(1) + 6$	0
2	$2(2)^2 - 8(2) + 6$	-2
3	$2(3)^2 - 8(3) + 6$	0
4	$2(4)^2 - 8(4) + 6$	6

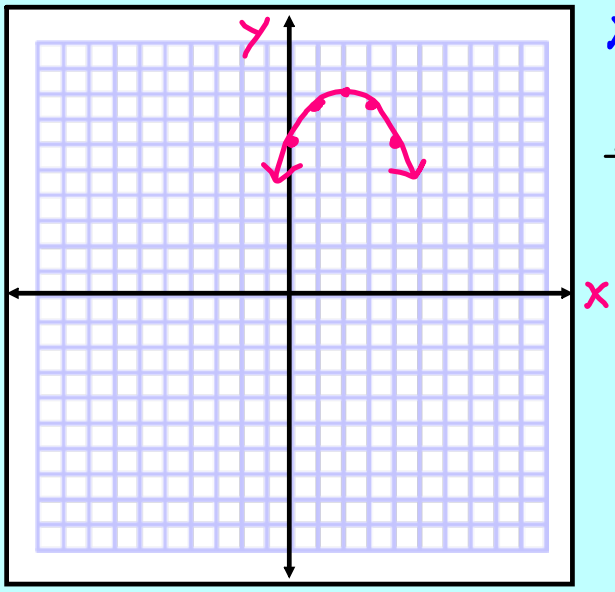
B. Graph Standard Form

Example: $g(x) = -\frac{1}{2}x^2 + 2x + 6$

$a = -\frac{1}{2}$ $b = 2$

a.o.s. $x = 2$

$x = \frac{-b}{2a} = \frac{-2}{2(-\frac{1}{2})} = \frac{-2}{-1} = 2$



x		y
0	$-\frac{1}{2}(0)^2 + 2(0) + 6$	6
1	$-\frac{1}{2}(1)^2 + 2(1) + 6$	$\frac{15}{2} = 7\frac{1}{2}$
2	$-\frac{1}{2}(2)^2 + 2(2) + 6$	8
3	$-\frac{1}{2}(3)^2 + 2(3) + 6$	$\frac{15}{2} = 7\frac{1}{2}$
4	$-\frac{1}{2}(4)^2 + 2(4) + 6$	6

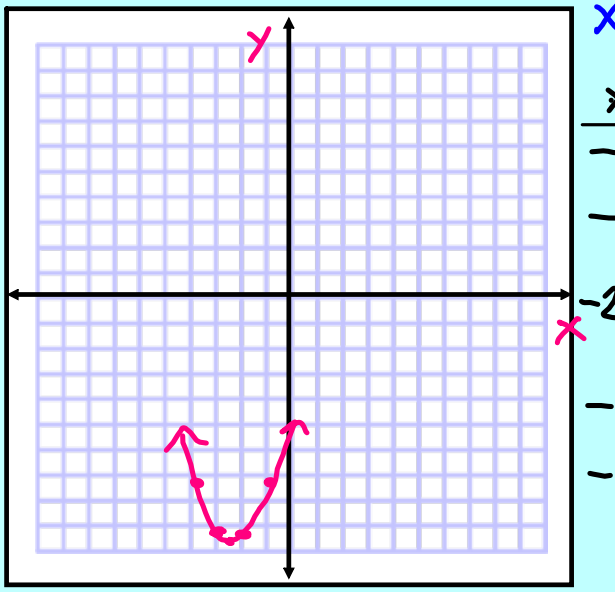
B. Graph Standard Form

Example: $f(x) = x^2 + 5x - 3$

$a = 1$ $b = 5$

a.o.s. $x = -2.5$

$x = \frac{-b}{2a} = \frac{-5}{2(1)} = \frac{-5}{2}$



x		y
−1	$(-1)^2 + 5(-1) - 3$ $1 - 5 - 3$	−7
−2	$(-2)^2 + 5(-2) - 3$ $4 - 10 - 3$	−9
−2.5	$(-2.5)^2 + 5(-2.5) - 3$ $\frac{25}{4} - \frac{50}{4} - \frac{12}{4}$	$-\frac{37}{4}$ / −9.25
−3	$(-3)^2 + 5(-3) - 3$ $9 - 15 - 3$	−9
−4	$(-4)^2 + 5(-4) - 3$ $16 - 20 - 3$	−7

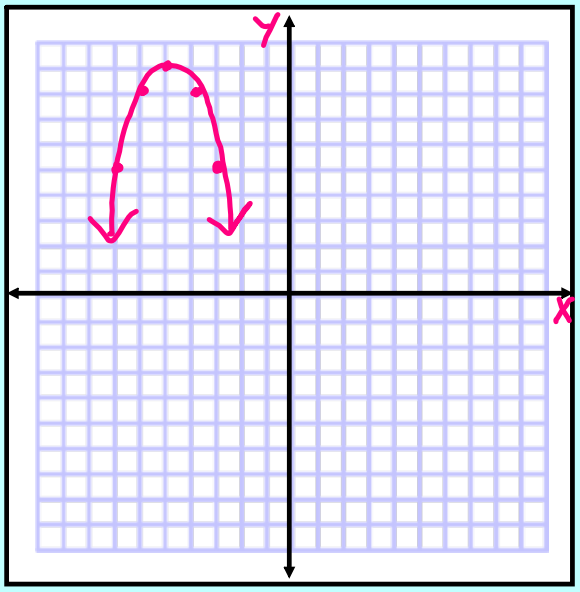
B. Graph Standard Form

Example: $f(x) = -x^2 - 10x - 16$

$a = -1 \quad b = -10$

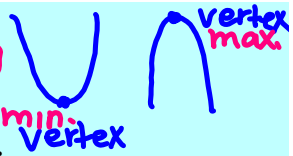
a.o.s. $x = -5$

$x = \frac{-b}{2a} = \frac{10}{2(-1)} = \frac{10}{-2} = -5$



x		y
-3	$-(-3)^2 - 10(-3) - 16$ $-9 + 30 - 16$	5
-4	$-(-4)^2 - 10(-4) - 16$ $-16 + 40 - 16$	8
-5	$-(-5)^2 - 10(-5) - 16$ $-25 + 50 - 16$	9
-6	$-(-6)^2 - 10(-6) - 16$ $-36 + 60 - 16$	8
-7	$-(-7)^2 - 10(-7) - 16$ $-49 + 70 - 16$	5

C. Find the ~~minimum or maximum value~~ JUST y-value



Example: Tell whether the function $y = 3x^2 - 18x + 20$ has a ~~minimum~~ value or a maximum value. Then find the minimum or maximum value.

$x = \frac{-b}{2a} = \frac{18}{2(3)} = \frac{18}{6} = 3$

$y = 3(3)^2 - 18(3) + 20$

$27 \quad -54 \quad +20$

$y = -7$

minimum = -7

Example: Follow the same directions with $y = -2x^2 + 4x + 3$.

$x = \frac{-b}{2a} = \frac{-4}{2(-2)} = \frac{-4}{-4} = 1$

$y = -2(1)^2 + 4(1) + 3$

$y = 5$

maximum = 5

Attachments

1-1 Standard Form of Quadratic Functions.doc