2.7 PIECEWISE FUNCTIONS

Combination of 2 or more functions

$$f(x) = \begin{cases} x + 2 & \text{if } x < 2 \\ 2x + 1 & \text{if } x \geq 2 \end{cases}$$

Example 1

Evaluate $f(x)$ when...

a) $x = 0$

$f(0) = 0 + 2$

$f(0) = 2$

b) $x = 2$

$f(2) = 2(2) + 1$

$f(2) = 4 + 1$

$f(2) = 5$

c) $x = 4$

$f(4) = 2(4) + 1$

$f(4) = 8 + 1$

$f(4) = 9$

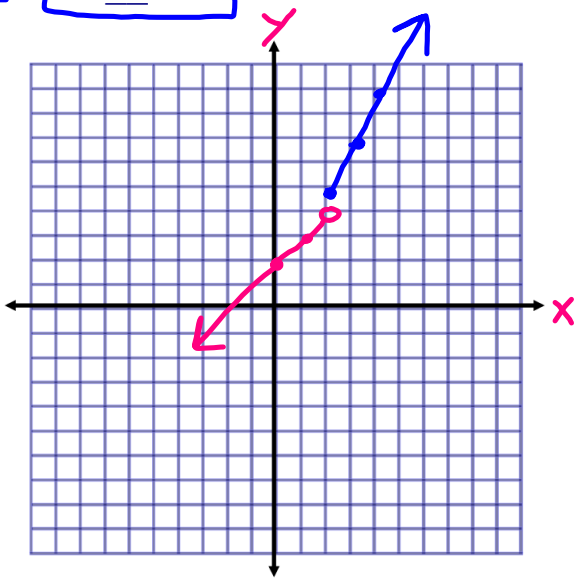
Example 2

Graph the function from Example 1.

$$f(x) = \begin{cases} x + 2 & \text{piece 1 if } x < 2 \\ 2x + 1 & \text{piece 2 if } x \geq 2 \end{cases}$$

x	y	
2	$2 + 2 = 4$	$\longrightarrow$ (2, 4) open
1	$1 + 2 = 3$	$\longrightarrow$ (1, 3)
0	$0 + 2 = 2$	$\longrightarrow$ (0, 2)
⋮		

x	y	
2	$2(2) + 1 = 5$	$\longrightarrow$ (2, 5) closed
3	$2(3) + 1 = 7$	$\longrightarrow$ (3, 7)
4	$2(4) + 1 = 9$	$\longrightarrow$ (4, 9)
⋮		



$$g(x) = \begin{cases} -3x + 2 & \text{if } x \le 3 \\ x - 10 & \text{if } x > 3 \end{cases}$$

Example 3

Evaluate $g(x)$ when...

a) $x = 0$

$g(0) = -3(0) + 2$

$g(0) = 0 + 2$

$g(0) = 2$

b) $x = 3$

$g(3) = -3(3) + 2$

$g(3) = -7$

c) $x = 9$

$g(9) = 9 - 10$

$g(9) = -1$

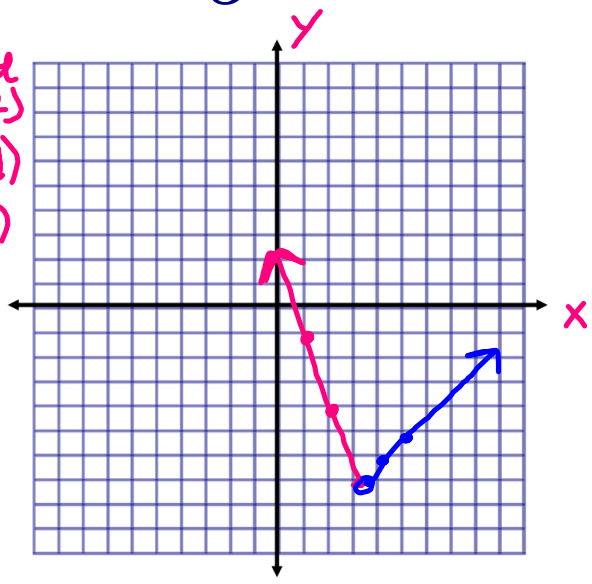
Example 4

Graph the function from Example 3.

$$g(x) = \begin{cases} -3x + 2 & \text{if } x \le 3 \quad \text{piece 1} \\ x - 10 & \text{if } x > 3 \quad \text{piece 2} \end{cases}$$

x	y	
3	$-3(3) + 2 = -7$	closed $(3, -7)$
2	$-3(2) + 2 = -4$	$(2, -4)$
1	$-3(1) + 2 = -1$	$(1, -1)$
⋮		

x	y	
3	$3 - 10 = -7$	open $(3, -7)$
4	$4 - 10 = -6$	$(4, -6)$
5	$5 - 10 = -5$	$(5, -5)$
⋮		



$$h(x) = \begin{cases} \frac{1}{2}x + 4 & \text{if } x > -2 \\ -x + 7 & \text{if } x \leq -2 \end{cases}$$

$6 > -2$ ✓

$-8 \leq -2$ ✓

$-2 \leq -2$ ✓

Example 5

Evaluate $h(x)$ when...

a) $x = -8$

$h(-8) = -(-8) + 7$

$h(-8) = 8 + 7$

$h(-8) = 15$

b) $x = -2$

$h(-2) = -(-2) + 7$

$h(-2) = 2 + 7$

$h(-2) = 9$

c) $x = 6$

$h(6) = \frac{1}{2}(6) + 4$

$h(6) = 3 + 4$

$h(6) = 7$

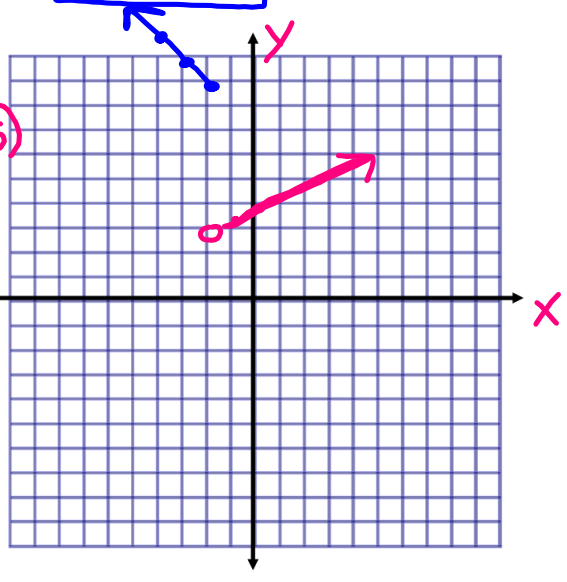
Example 6

Graph the function from Example 5.

$$h(x) = \begin{cases} \frac{1}{2}x + 4 & \text{if } x > -2 \\ -x + 7 & \text{if } x \leq -2 \end{cases}$$

x	y	
−2	$\frac{1}{2}(-2)+4 = 3$	$\to (-2, 3)$ open
−1	$\frac{1}{2}(-1)+4 = 3.5$	$\to (-1, 3.5)$
0	$\frac{1}{2}(0)+4 = 4$	$\to (0, 4)$
⋮		

x	y	
−2	$-(-2)+7 = 9$	$\to (-2, 9)$ closed
−3	$-(-3)+7 = 10$	$\to (-3, 10)$
−4	$-(-4)+7 = 11$	$\to (-4, 11)$
⋮		



Example 7

Graph the function below.

$$k(x) = \begin{cases} 3x & \text{if } 0 \le x < 2 \\ 6 & \text{if } 2 < x \le 4 \\ -x + 10 & \text{if } 4 < x \le 6 \end{cases}$$

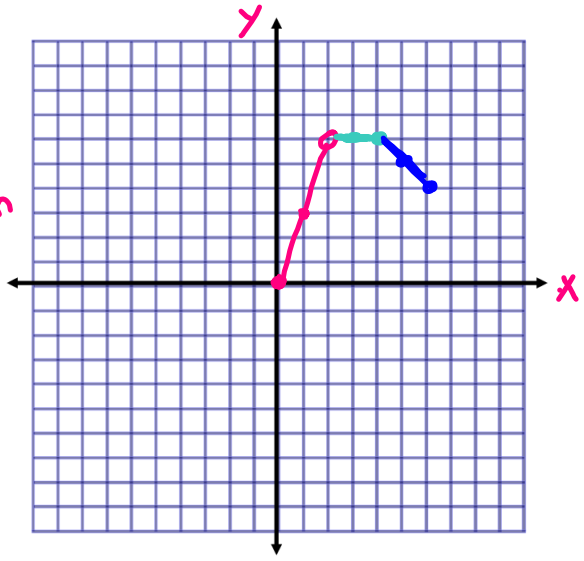
piece 1 piece 2 piece 3

piece 1

x	y
0	3(0) = 0 → (0, 0)
1	3(1) = 3 → (1, 3)
2	3(2) = 6 → (2, 6) open

x	y
2	6
3	6
4	6

x	y
4	-4 + 10 = 6 → (4, 6) open
5	-5 + 10 = 5 → (5, 5)
6	-6 + 10 = 4 → (6, 4) closed



Example 8

Graph the function below.

$$w(x) = \begin{cases} -2x - 3 & \text{if } -6 \le x < 0 \\ x - 2 & \text{if } 0 \le x < 4 \\ -\frac{1}{4}x + 3 & \text{if } 4 \le x \le 8 \end{cases}$$