2.2 Graphs of Functions

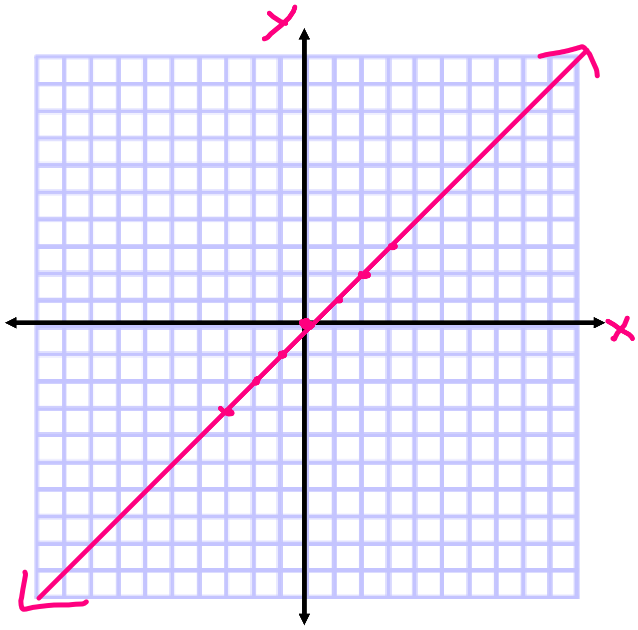
Parent Functions: the simplest form of the function

In this chapter we will begin to "transform" the parent functions and describe the transformations.

Make a table using x = −3, −2, −1, 0, 1, 2, & 3.
Then graph.

$y = x$

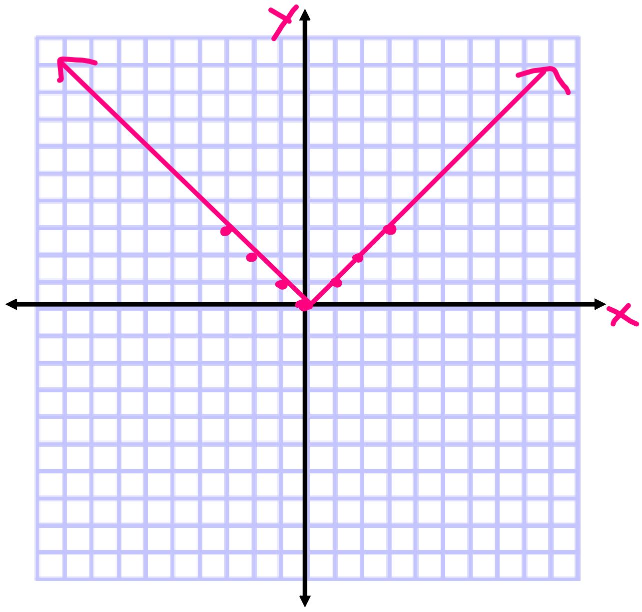
x	y
−3	−3
−2	−2
−1	−1
0	0
1	1
2	2
3	3



Make a table using x = −3, −2, −1, 0, 1, 2, & 3.
Then graph.

$y = |x|$

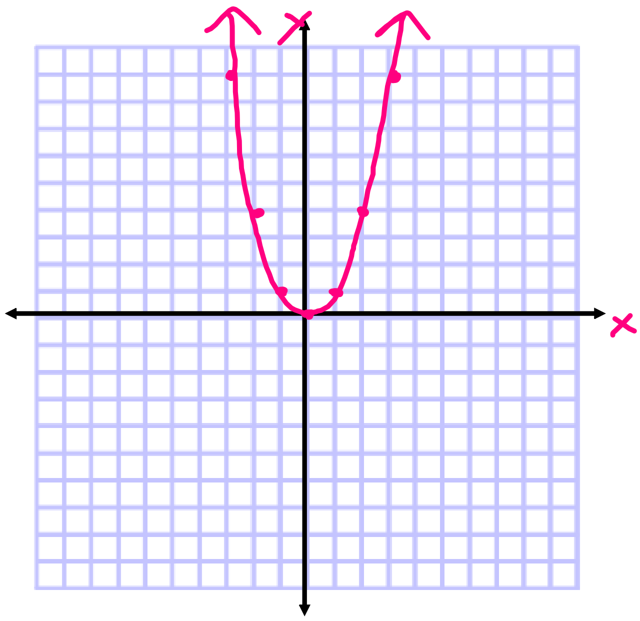
x	y
−3	3
−2	2
−1	1
0	0
1	1
2	2
3	3



Make a table using x = −3, −2, −1, 0, 1, 2, & 3.
Then graph.

$y = x^2$

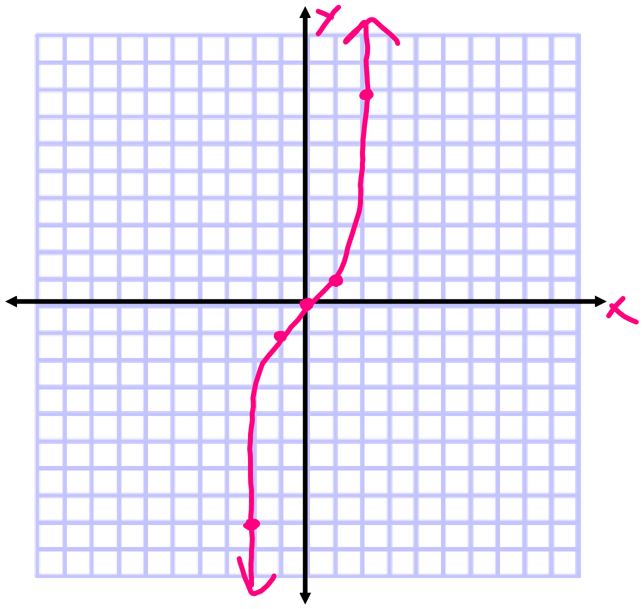
x	y
−3	9
−2	4
−1	1
0	0
1	1
2	4
3	9



Make a table using x = −3, −2, −1, 0, 1, 2, & 3.
Then graph.

$y = x^3$

x	y
−3	−27
−2	−8
−1	−1
0	0
1	1
2	8
3	27

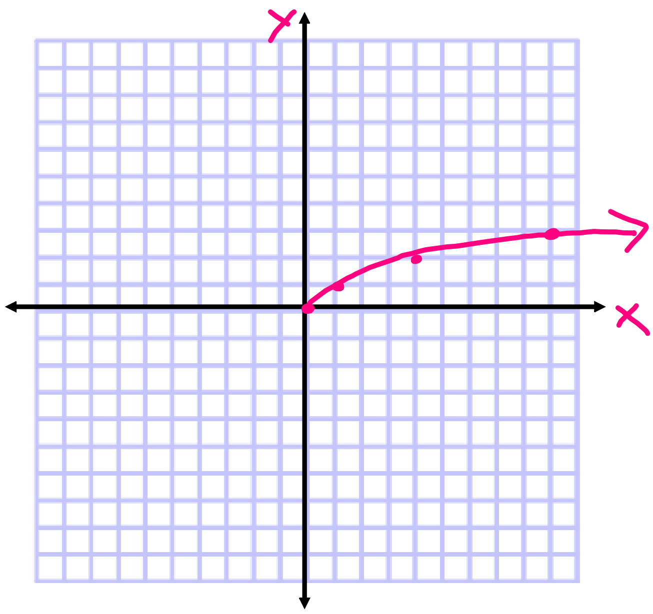


Make a table using x = −9, −4, −1, 0, 1, 4, & 9.
Then graph.

$y = \sqrt{x}$

$x \geq 0$

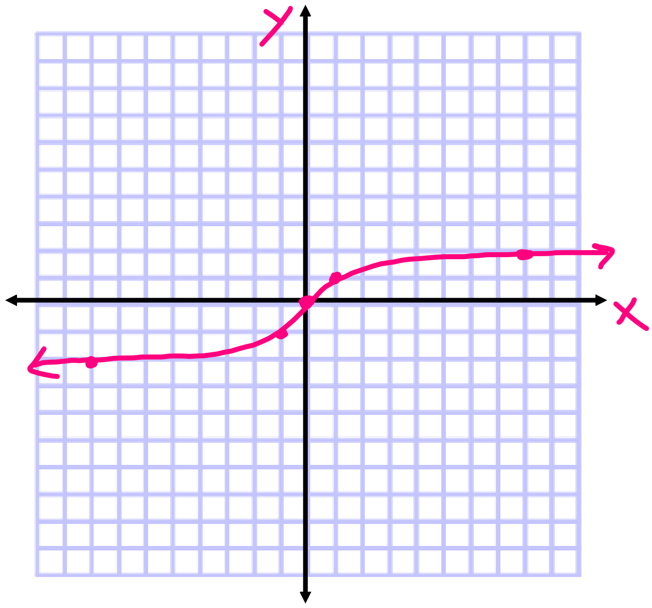
x	y
~~−9~~	~~3i~~
~~−4~~	~~2i~~
~~−1~~	~~i~~
0	0
1	1
4	2
9	3



Make a table using x = −27, −8, −1, 0, 1, 8, & 27. Then graph.

$y = \sqrt[3]{x}$

x	y
−27	−3
−8	−2
−1	−1
0	0
1	1
8	2
27	3

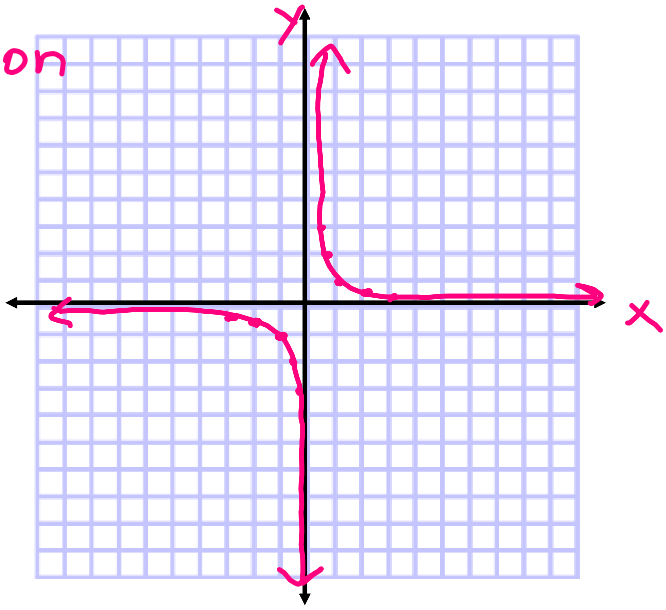


Make a table using x = −3, −2, −1, $-\frac{1}{2}$, $-\frac{1}{3}$, 0, $\frac{1}{3}$, $\frac{1}{2}$, 1, 2, & 3. Then graph.

reciprocal function

$y = \frac{1}{x}$ $x \neq 0$

x	y
−3	−1/3
−2	−1/2
−1	−1
−1/2	−2
−1/3	−3
~~0~~	
1/3	3
1/2	2
1	1
2	1/2
3	1/3

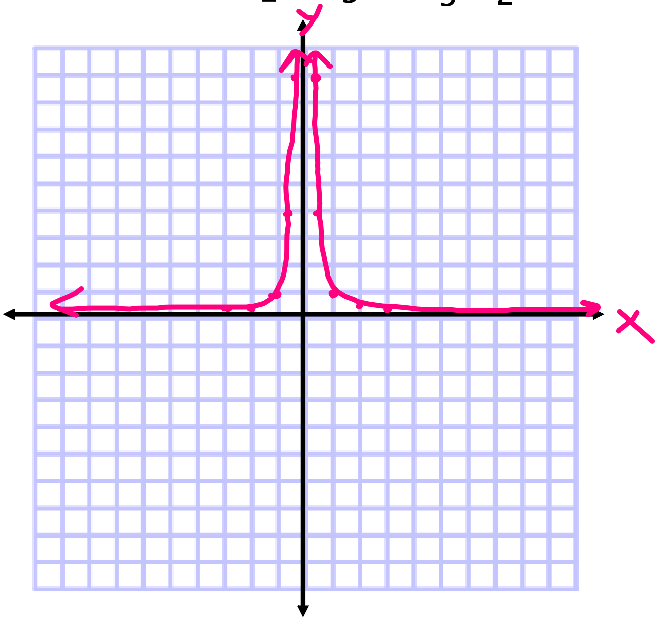


Make a table using x = −3, −2, −1, $-\frac{1}{2}$, $-\frac{1}{3}$, 0, $\frac{1}{3}$, $\frac{1}{2}$, 1, 2, & 3. Then graph.

reciprocal function

$$y = \frac{1}{x^2}$$

x	y
−3	1/9
−2	1/4
−1	1
−1/2	4
−1/3	9
0	
1/3	9
1/2	4
1	1
2	1/4
3	1/9



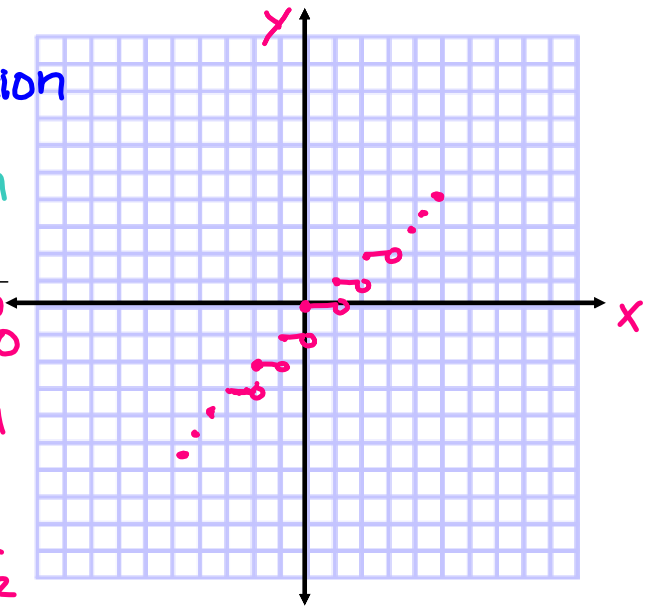
Make a table using x = −3, −2.7, −2.3, −2, −1.7, −1.3, −1, −0.7, −0.3, 0, 0.3, 0.7, 1, 1.3, 1.7, 2, 2.3, 2.7, & 3. Then graph.

rounding down function

y = [x] step function

x	y
−3	[−3] = −3
−2.7	[−2.7] = −3
−2.3	[−2.3] = −3
−2	[−2] = −2
−1.7	[−1.7] = −2
−1.3	[−1.3] = −2
−1	[−1] = −1
−0.7	[−0.7] = −1
−0.3	[−0.3] = −1
0	[0] = 0

x	y
0.3	[0.3] = 0
0.7	[0.7] = 0
1	[1] = 1
1.3	[1.3] = 1
1.7	[1.7] = 1
2	[2] = 2
2.3	[2.3] = 2
2.7	[2.7] = 2
3	[3] = 3

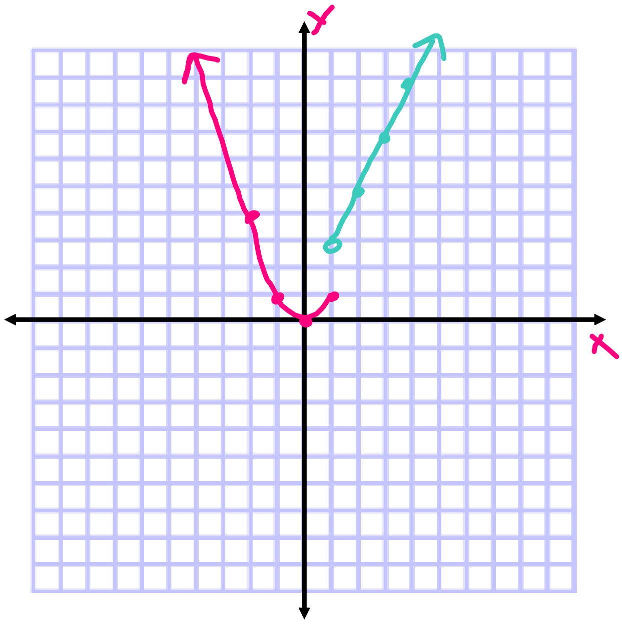


Sketch the graph of the piecewise function.

$$f(x) = \begin{cases} x^2 \text{ piece1 if } \boxed{x \le 1}^{\text{domain}} \\ 2x + 1 \text{ if } x > 1 \text{ piece 2} \end{cases}$$

x	y
closed 1	1
0	0
−1	1
−2	4

x	y
open 1	2(1)+1=3
2	2(2)+1=5
3	2(3)+1=7
4	2(4)+1=9



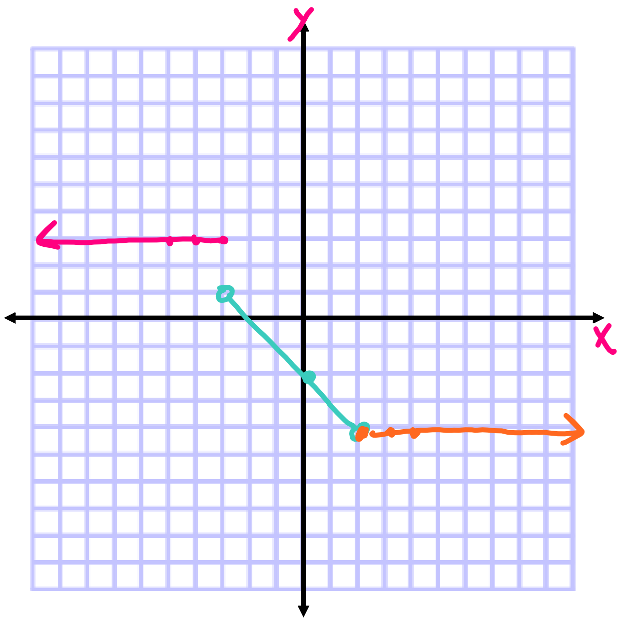
Sketch the graph of the piecewise function.

$$f(x) = \begin{cases} 3 \text{ piece1 if } x \le -3 \\ -x - 2 \text{ if } -3 < x < 2 \text{ piece 2} \\ -4 \text{ piece3 if } x \ge 2 \end{cases}$$

x	y
closed −3	3
−4	3
−5	3

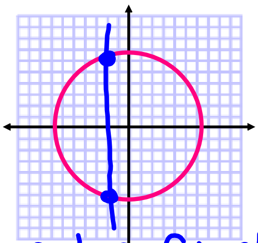
x	y
open −3	1
0	−2
open 2	−4

x	y
closed 2	−4
3	−4
4	−4

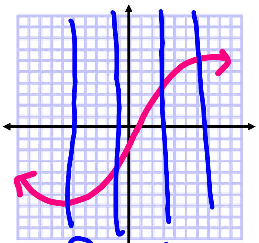


Vertical Line Test: If a vertical line crosses the graph at 2 or more points (at the same time), then the graph is not a function.

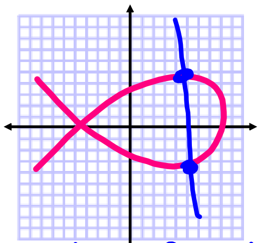
Determine if the following graphs represent functions.



not a function

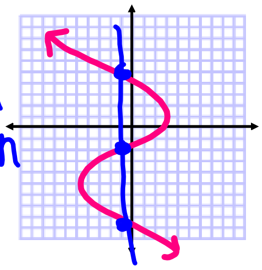


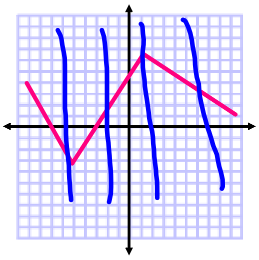
function



not a function

not a function





function

Any equation in the variables x and y defines a relationship between these variables.

For example, the equation

$$y - x^2 = 0$$

defines a relationship between x and y.

Does this equation define y as a function of x?

To find out, solve for y.

$$\begin{aligned} y - x^2 &= 0 \\ +x^2 \quad & +x^2 \\ \hline y &= x^2 \end{aligned}$$

function

The equation defines a rule, or function, that gives one value of y for each value of x.

We can express this in function notation as $f(x) = x^2$.

However, not every equation defines y as a function of x.

Does each equation define y as a function of x?

$$y - x^2 = 2$$
$$+x^2 \quad +x^2$$
$$y = x^2 + 2$$

function

$$f(x) = x^2 + 2$$

$$x^2 + y^2 = 4$$
$$-x^2 \qquad -x^2$$
$$\sqrt{y^2} = \sqrt{4 - x^2}$$
$$y = \pm\sqrt{4 - x^2}$$

not a function