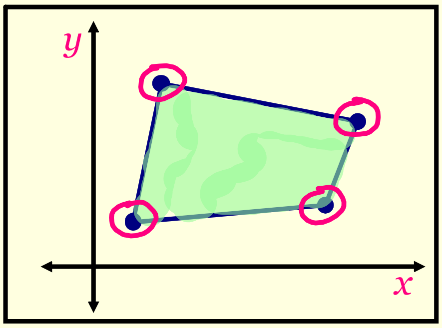
3.5 LINEAR PROGRAMMING & OPTIMIZATION

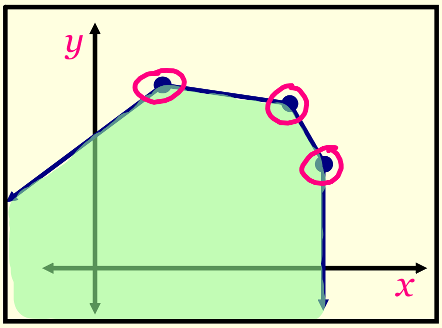
- Used to find optimal solutions
- Include the following characteristics:
  - The inequalities contained in the problem are called constraints.
  - The solution to the set of constraints is called the feasible region.
  - The function to be maximized or minimized is called the objective function.

CORNER-POINT PRINCIPLE

In linear programming, the maximum and minimum values of the objective function each occur at one of the vertices of the feasible region.



Bounded Region



Unbounded Region

EXAMPLE 1

A. Graph the feasible region.

B. Find the maximum and minimum values, if they exist.

$P = 3x + y$ ← objective function

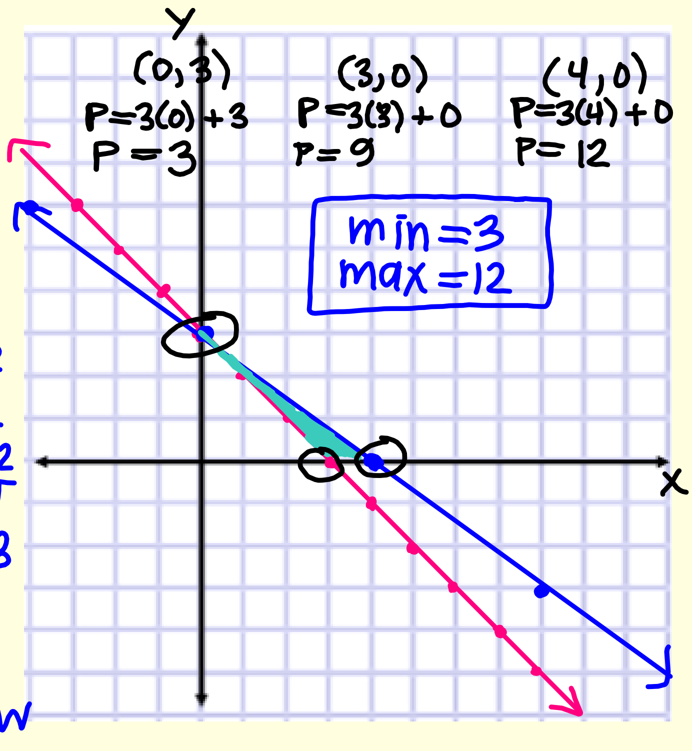
Constraints:

$$\begin{cases} x + y \geq 3 \\ 3x + 4y \leq 12 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

QI

$x + y \geq 3$
$-x \quad -x$
$y \geq -x + 3$
$m = -1$
y-int $= 3$
shade above

$3x + 4y \leq 12$
$-3x \quad -3x$
$\frac{4y}{4} \leq \frac{-3x}{4} + \frac{12}{4}$
$y \leq -\frac{3}{4}x + 3$
$m = -\frac{3}{4}$
y-int $= 3$
shade below



EXAMPLE 2

A. Graph the feasible region.

B. Find the maximum and minimum values, if they exist.

$E = x + y$

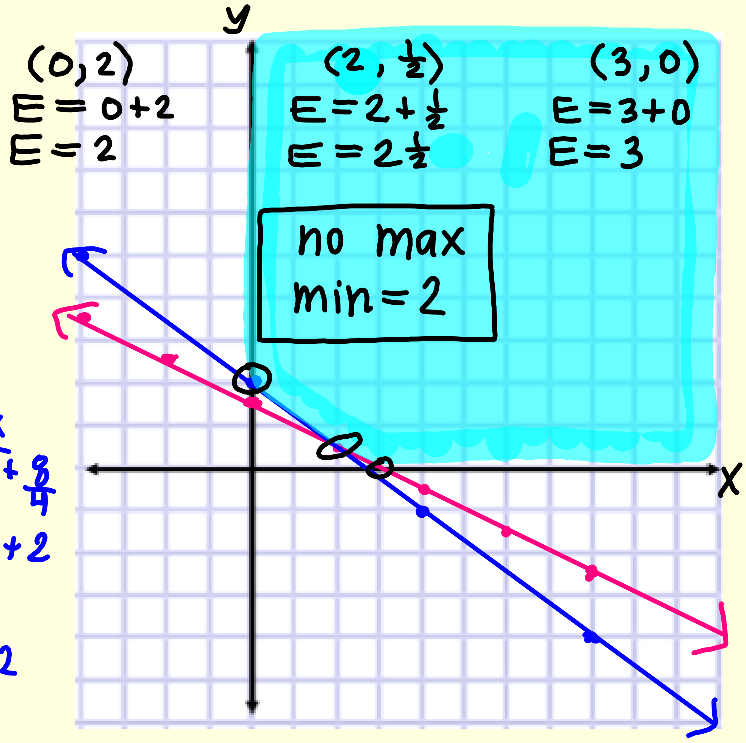
Constraints:

$$\begin{cases} x + 2y \geq 3 \\ 3x + 4y \geq 8 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

QI

$x + 2y \geq 3$
$-x \quad -x$
$\frac{2y}{2} \geq \frac{-x}{2} + \frac{3}{2}$
$y \geq -\frac{1}{2}x + \frac{3}{2}$
$m = -\frac{1}{2}$
y-int $= \frac{3}{2}$
above

$3x + 4y \geq 8$
$-3x \quad -3x$
$\frac{4y}{4} \geq \frac{-3x}{4} + \frac{8}{4}$
$y \geq -\frac{3}{4}x + 2$
$m = -\frac{3}{4}$
y-int $= 2$
above



EXAMPLE 3

A. Graph the feasible region.
B. Find the maximum and minimum values, if they exist.

$M = 3x + 2y$

Constraints:

$$\begin{cases} x + y \le 5 \\ y - x \ge 5 \\ 4x + y \ge -10 \end{cases}$$

$x+y \le 5$
$-x \quad -x$
$y \le -x+5$ — $m=-1$, y-int$=5$, shade below

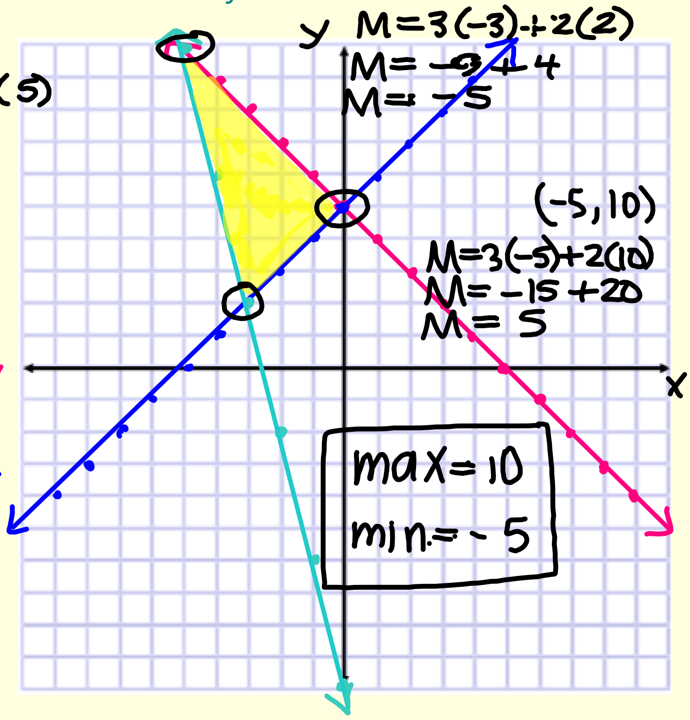
$y-x \ge 5$
$+x \quad +x$
$y \ge x+5$ — $m=1$, y-int$=5$, shade above

$4x+y \ge -10$
$-4x \quad -4x$
$y \ge -4x-10$ — $m=-4$, y-int$=-10$, shade above

(0,5)
$M=3(0)+2(5)$
$M=0+10$
$M=10$

(-3,2)
$M=3(-3)+2(2)$
$M=-9+4$
$M=-5$

(-5,10)
$M=3(-5)+2(10)$
$M=-15+20$
$M=5$



max = 10
min = -5

EXAMPLE 4

A. Graph the feasible region.
B. Find the maximum and minimum values, if they exist.

$K = 5x + y$

Constraints:

$$\begin{cases} 2x + y \le 3 \\ -3x + y \le 3 \\ y \ge -3 \end{cases}$$

hor. shade above

$2x+y \le 3$
$-2x \quad -2x$
$y \le -2x+3$ — $m=-2$, y-int$=3$, shade below

$-3x+y \le 3$
$+3x \quad +3x$
$y \le 3x+3$ — $m=3$, y-int$=3$, shade below

(0,3)
$K=5(0)+3$
$K=3$

(-2,-3)
$K=5(-2)+-3$
$K=-13$

(3,-3)
$K=5(3)+-3$
$K=12$

max = 12
min = -13