2.1 Functions

Relation: a set of ordered pairs
first number in pair = domain / x-values
second number in pair = range / y-values

Function: a rule that assigns to each element x in set A exactly one element, called $f(x)$, in a set B domain <u>cannot</u> repeat

$f(x)$ is read "f of x" or "f at x"

Set A is the domain or independent variable
Set B is the range or dependent variable

Example 1: $f(x) = x^2$

a) Evaluate $f(3)$, $f(-2)$, and $f(\sqrt{5})$.

$f(3) = (3)^2$ $\quad f(-2) = (-2)^2$ $\quad f(\sqrt{5}) = (\sqrt{5})^2$

$\boxed{f(3) = 9}$ $\quad \boxed{f(-2) = 4}$ $\quad \boxed{f(\sqrt{5}) = 5}$

b) Find the domain and range of f.

D: $\{3, -2, \sqrt{5}\}$
R: $\{9, 4, 5\}$

Example 2: $f(x) = 3x^2 + x - 5$

Evaluate each function value.

a) $f(-2) = 3(-2)^2 + (-2) - 5$
$= 3(4) + -2 - 5$
$= 12 + -2 - 5$
$\boxed{f(-2) = 5}$

d) $f(\tfrac{1}{2}) = 3(\tfrac{1}{2})^2 + (\tfrac{1}{2}) - 5$
$= 3(\tfrac{1}{4}) + (\tfrac{1}{2}) - 5$
$= \tfrac{3}{4} + \tfrac{2}{4} - \tfrac{20}{4}$
$\boxed{f(\tfrac{1}{2}) = -\tfrac{15}{4}}$

b) $f(0) = 3(0)^2 + (0) - 5$
$= 0 + 0 - 5$
$\boxed{f(0) = -5}$

e) $\boxed{f(a) = 3a^2 + a - 5}$

c) $f(4) = 3(4)^2 + (4) - 5$
$= 48 + 4 - 5$
$\boxed{f(4) = 47}$

A piecewise function is a function defined by at least two equations ("pieces"), each of which applies to a different part of the domain.

Example 3: $f(x) = \begin{cases} 1 - x & \text{if } x \le 1 \\ x^2 & \text{if } x > 1 \end{cases}$ $x = -2$ $x = 1$ $x = 2$ $x = 5$

Evaluate at $x = -2, 1, 2,$ and 5.

$f(-2) = 1 - (-2)$
$\boxed{f(-2) = 3}$

$f(1) = 1 - 1$
$\boxed{f(1) = 0}$

$f(2) = (2)^2$
$\boxed{f(2) = 4}$

$f(5) = (5)^2$
$\boxed{f(5) = 25}$

Example 4: $f(x) = 2x^2 + 3x - 1$

Evaluate the following.

a) $\boxed{f(a) = 2a^2 + 3a - 1}$

b) $f(-a) = 2(-a)^2 + 3(-a) - 1$

$\boxed{f(-a) = 2a^2 - 3a - 1}$

c) $f(a + h) = 2\overbrace{(a+h)^2}^{(a+h)(a+h)} + 3(a+h) - 1$

$= 2(a^2 + 2ah + h^2) + 3(a+h) - 1$

$\boxed{f(a+h) = 2a^2 + 4ah + 2h^2 + 3a + 3h - 1}$

d) $\dfrac{f(a+h) - f(a)}{h}$; $h \neq 0$

$\dfrac{(\cancel{2a^2} + 4ah + 2h^2 + \cancel{3a} + 3h \cancel{-1}) + (\cancel{-2a^2} \cancel{-3a} \cancel{+1})}{h}$

$\dfrac{4ah + 2h^2 + 3h}{h} \rightarrow \dfrac{\cancel{h}(4a + 2h + 3)}{\cancel{h}} = \boxed{4a + 2h + 3}$

Recall that the domain of a function is the set of all inputs for the function. The domain may be stated explicitly.

Ex: $f(x) = x^2$, $\overset{\text{specific domain}}{\underline{0 \le x \le 5}}$

Domain: $[0, 5]$

If the domain is not explicitly stated, then it is *the set of all real numbers for which the expression is defined as a real number*.

Example 5: Find the domain.

a) $f(x) = \frac{1}{x - 4}$ can't $= 0$

$x - 4 \neq 0$
$+4 \quad +4$
$x \neq 4$

Domain: $(-\infty, 4) \cup (4, \infty)$

b) $g(x) = \sqrt{x + 2}$ can't be negative

$x + 2 \geq 0$
$-2 \quad -2$
$x \geq -2$

Domain: $[-2, \infty)$

Example 6: Find the domain.

a) $k(x) = \frac{1}{x^2 - x}$ can't equal 0

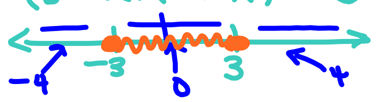
Domain: $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

$x^2 - x \neq 0$
$x(x - 1) \neq 0$
$x \neq 0 \qquad x - 1 \neq 0$
$+1 \quad +1$
$x \neq 1$

b) $h(x) = \sqrt{9 - x^2}$ can't be negative

Domain: $[-3, 3]$

$9 - x^2 \geq 0$
$(3 - x)(3 + x) \geq 0$



$(3 - -4)(3 + -4)$
$(7)(-1)$

$(3 - 0)(3 + 0)$
$(3)(3)$

$(3 - 4)(3 + 4)$
$(-1)(7)$

Example 7: Find the domain.

a) $f(t) = \dfrac{1}{\sqrt{\boxed{t+1}}}$ can't = 0 & can't be neg.

Domain: $(-1, \infty)$

$$\begin{array}{r} t+1>0 \\ -1\quad -1 \\ \hline t>-1 \end{array}$$

b) $j(x) = 3x^2 + 4x - 9$

no restrictions

Domain: $(-\infty, \infty)$

FOUR WAYS TO REPRESENT A FUNCTION

VERBAL

Using words:

$P(t)$ is "the population of the world at time t"

Relation of population P and time t

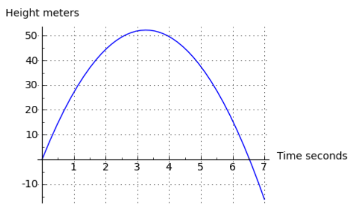
ALGEBRAIC

Using a formula:

$A(r) = \pi r^2$

Area of a circle

VISUAL



Graph of the position of an object that has been thrown upward

NUMERICAL

Using a table of values:

w (ounces)	$C(w)$ (dollars)
$0 < w \le 1$	0.34
$1 < w \le 2$	0.57
$2 < w \le 3$	0.80
.	.
.	.
.	.

Cost of mailing a first-class letter