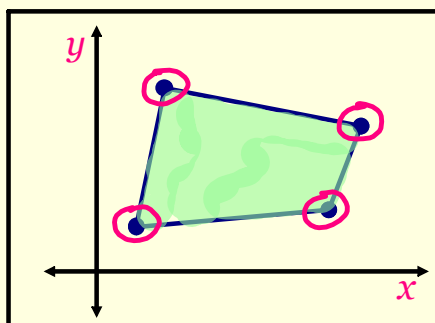


3.5 LINEAR PROGRAMMING & OPTIMIZATION

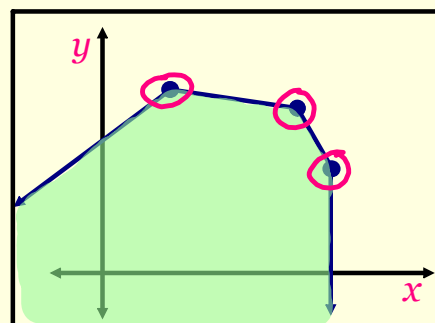
- Used to find **optimal** solutions
- Include the following characteristics:
 - The inequalities contained in the problem are called constraints.
 - The solution to the set of constraints is called the feasible region.
 - The function to be maximized or minimized is called the objective function.

CORNER-POINT PRINCIPLE

In linear programming, the maximum and minimum values of the objective function each occur at one of the vertices of the feasible region.



Bounded Region



Unbounded Region

EXAMPLE 1

- A. Graph the feasible region.
- B. Find the maximum and minimum values, if they exist.

$P = 3x + y$ ← objective function

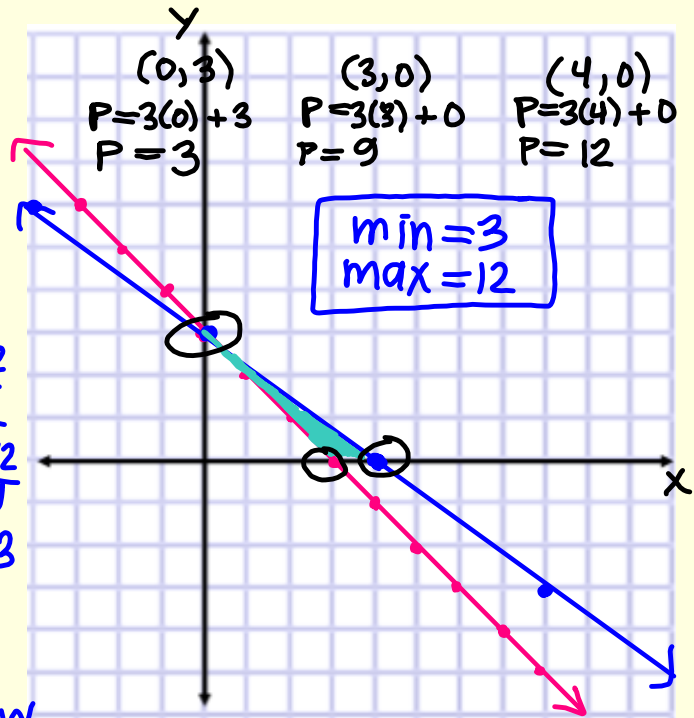
Constraints:

$$\begin{cases} x + y \geq 3 \\ 3x + 4y \leq 12 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

QI

$$\begin{array}{r} x + y \geq 3 \\ -x \quad -x \\ \hline y \geq -x + 3 \\ m = -1 \\ y\text{-int} = 3 \\ \text{shade above} \end{array}$$

$$\begin{array}{r} 3x + 4y \leq 12 \\ -3x \quad -3x \\ \hline 4y \leq -3x + 12 \\ \frac{4y}{4} \leq \frac{-3x + 12}{4} \\ y \leq -\frac{3}{4}x + 3 \\ m = -\frac{3}{4} \\ y\text{-int} = 3 \\ \text{shade below} \end{array}$$



EXAMPLE 2

- A. Graph the feasible region.
- B. Find the maximum and minimum values, if they exist.

$E = x + y$

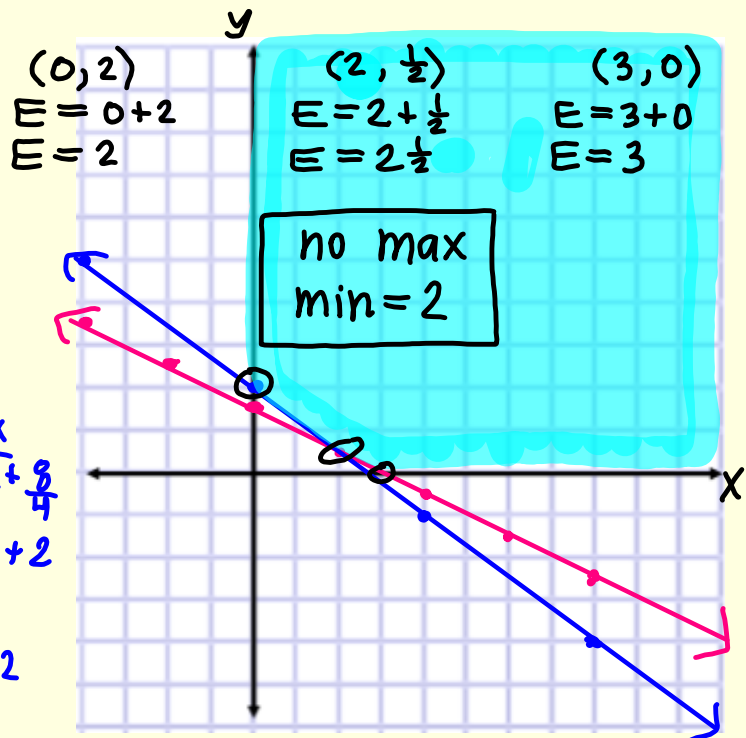
Constraints:

$$\begin{cases} x + 2y \geq 3 \\ 3x + 4y \geq 8 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

QI

$$\begin{array}{r} x + 2y \geq 3 \\ -x \quad -x \\ \hline 2y \geq -x + 3 \\ \frac{2y}{2} \geq \frac{-x + 3}{2} \\ y \geq -\frac{1}{2}x + \frac{3}{2} \\ m = -\frac{1}{2} \\ y\text{-int} = \frac{3}{2} \\ \text{above} \end{array}$$

$$\begin{array}{r} 3x + 4y \geq 8 \\ -3x \quad -3x \\ \hline 4y \geq -3x + 8 \\ \frac{4y}{4} \geq \frac{-3x + 8}{4} \\ y \geq -\frac{3}{4}x + 2 \\ m = -\frac{3}{4} \\ y\text{-int} = 2 \\ \text{above} \end{array}$$



EXAMPLE 3

$M = 3x + 2y$

Constraints:

$$\begin{cases} x + y \leq 5 \\ y - x \geq 5 \\ 4x + y \geq -10 \end{cases}$$

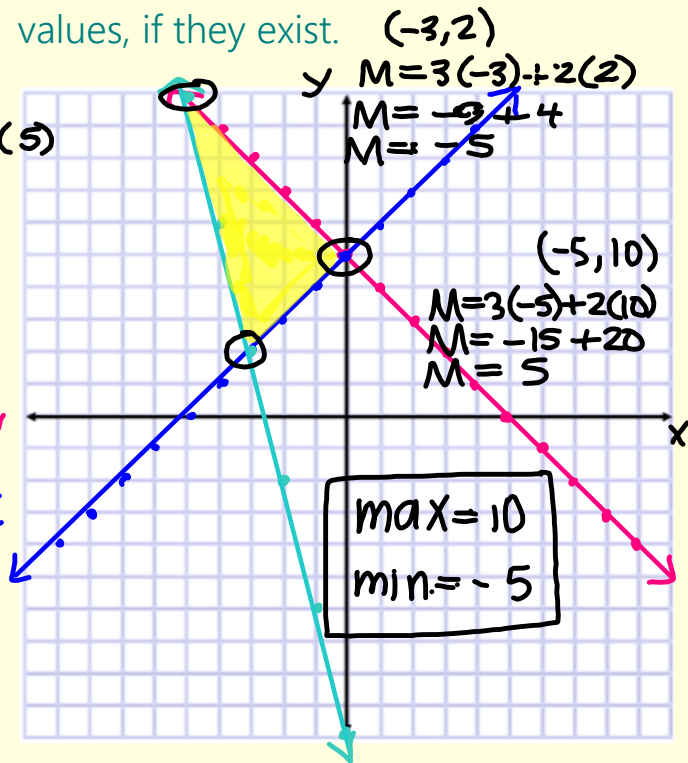
$\frac{x+y \leq 5}{-x \quad -x}$
 $y \leq -x + 5$ $m = -1$ $y\text{-int} = 5$ *shade below*

$\frac{y-x \geq 5}{+x \quad +x}$
 $y \geq x + 5$ $m = 1$ $y\text{-int} = 5$ *shade above*

$\frac{4x+y \geq -10}{-4x \quad -4x}$
 $y \geq -4x - 10$ $m = -4$ $y\text{-int} = -10$ *shade above*

$(0, 5)$
 $M = 3(0) + 2(5)$
 $M = 0 + 10$
 $M = 10$

- A. Graph the feasible region.
 B. Find the maximum and minimum values, if they exist.



EXAMPLE 4

$K = 5x + y$

Constraints:

$$\begin{cases} 2x + y \leq 3 \\ -3x + y \leq 3 \\ y \geq -3 \end{cases}$$

- A. Graph the feasible region.
 B. Find the maximum and minimum values, if they exist.

