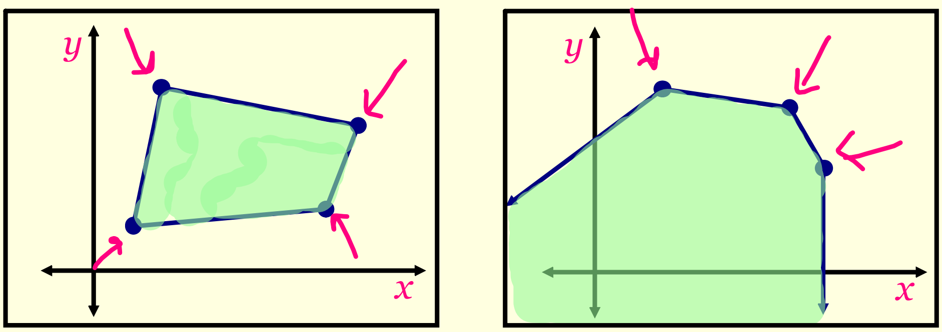
3.5 LINEAR PROGRAMMING & OPTIMIZATION

- Used to find optimal solutions
- Include the following characteristics:
  - The inequalities contained in the problem are called constraints.
  - The solution to the set of constraints is called the feasible region.
  - The function to be maximized or minimized is called the objective function.

CORNER-POINT PRINCIPLE

In linear programming, the maximum and minimum values of the objective function each occur at one of the vertices of the feasible region.



Bounded Region *Unbounded Region*

EXAMPLE 1

objective function

P = 3x + y

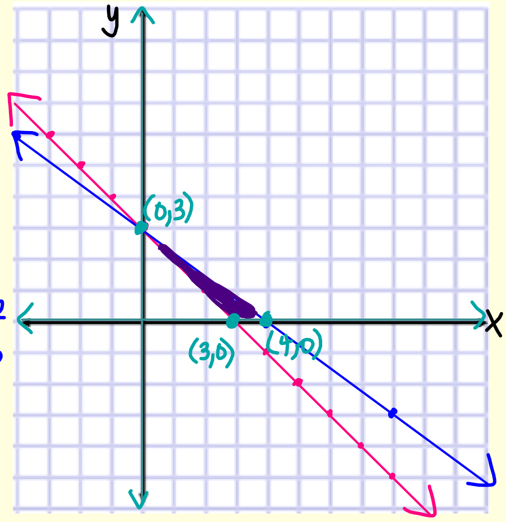
Constraints:

$$\begin{cases} x + y \geq 3 \\ 3x + 4y \leq 12 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

$x \geq 0$, $y \geq 0$ } Quad. 1

A. Graph the feasible region.

B. Find the maximum and minimum values, if they exist.



$x + y \geq 3$
$-x \quad -x$
$y \geq -x + 3$
$m = -1$
$b = 3$
solid
shade above

$3x + 4y \leq 12$
$4y \leq -3x + 12$
$y \leq -\frac{3}{4}x + 3$
$m = -\frac{3}{4}$
$b = 3$
solid
shade below

$P = 3x + y$

(0,3)	(3,0)	(4,0)
$P = 3(0) + 3$	$P = 3(3) + 0$	$P = 3(4) + 0$
$P = 3$	$P = 9$	$P = 12$
minimum		maximum

EXAMPLE 2

E = x + y

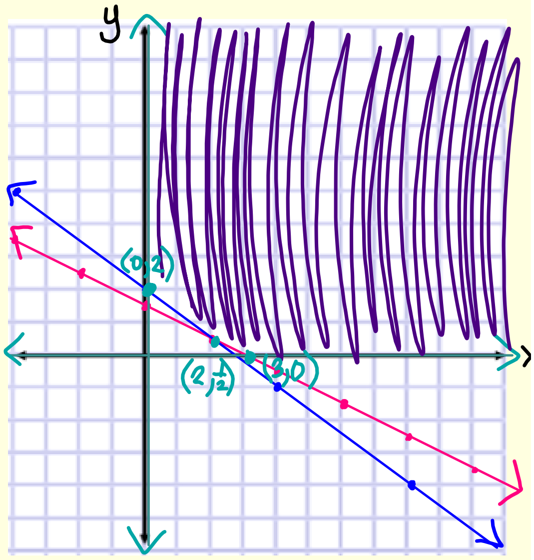
Constraints:

$$\begin{cases} x + 2y \geq 3 \\ 3x + 4y \geq 8 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

$x \geq 0$, $y \geq 0$ } Quad 1

A. Graph the feasible region.

B. Find the maximum and minimum values, if they exist.



$x + 2y \geq 3$
$2y \geq -x + 3$
$y \geq -\frac{1}{2}x + \frac{3}{2}$
$m = -\frac{1}{2}$
$b = \frac{3}{2}$
solid
shade ↑

$3x + 4y \geq 8$
$4y \geq -3x + 8$
$y \geq -\frac{3}{4}x + 2$
$m = -\frac{3}{4}$
$b = 2$
solid
shade ↑

$E = x + y$

no maximum

(0,2)	$(2,\frac{1}{2})$	(3,0)
$E = 0 + 2$	$E = 2 + \frac{1}{2}$	$E = 3 + 0$
$E = 2$	$E = 2\frac{1}{2}$	$E = 3$
minimum		

EXAMPLE 3

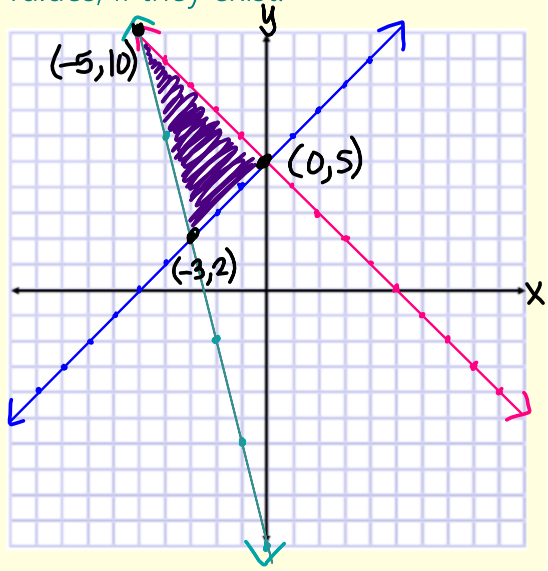
A. Graph the feasible region.
B. Find the maximum and minimum values, if they exist.

$M = 3x + 2y$

Constraints:

$$\begin{cases} x + y \le 5 \\ y - x \ge 5 \\ 4x + y \ge -10 \end{cases}$$

$x+y\le 5$	$y-x\ge 5$	$4x+y\ge -10$
$y\le -x+5$	$y\ge x+5$	$y\ge -4x-10$
$m=-1$	$m=1$	$m=-4$
$b=5$	$b=5$	$b=-10$
solid	solid	solid
shade ↓	shade ↑	shade ↑



$M=3x+2y$

(0,5)	(-3,2)	(-5,10)
$M=3(0)+2(5)$	$M=3(-3)+2(2)$	$M=3(-5)+2(10)$
$M=10$	$M=-5$	$M=5$
maximum	minimum	

EXAMPLE 4

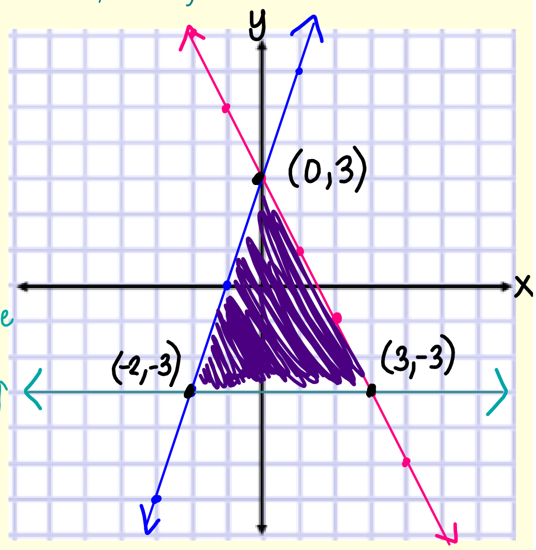
A. Graph the feasible region.
B. Find the maximum and minimum values, if they exist.

$K = 5x + y$

Constraints:

$$\begin{cases} 2x + y \le 3 \\ -3x + y \le 3 \\ y \ge -3 \end{cases}$$

$2x+y\le 3$	$-3x+y\le 3$	$y\ge -3$
$y\le -2x+3$	$y\le 3x+3$	hor. line
$m=-2$	$m=3$	solid
$b=3$	$b=3$	shade ↑
solid	solid	
shade ↓	shade ↓	



$K=5x+y$

(0,3)	(-2,-3)	(3,-3)
$K=5(0)+3$	$K=5(-2)+(-3)$	$K=5(3)+(-3)$
$K=3$	$K=-13$	$K=12$
	minimum	maximum

EXAMPLE 5 $x = $ # of afghans $y = $ # of sweaters

A small company produces knitted afghans and sweaters and sells them through a chain of specialty stores. The company is to supply the stores with a total of no more than 100 afghans and sweaters per day. The stores guarantee that they will sell at least 10 and no more than 60 afghans per day and at least 20 sweaters per day. The company makes a profit of \$10 on each afghan and a profit of \$12 on each sweater.

A. Write a system of inequalities to represent the constraints.

$$\begin{cases} x + y \le 100 \\ 10 \le x \le 60 \\ y \ge 20 \end{cases}$$

B. Write an objective function for the company's total profit, P, from the sales of afghans and sweaters.

$$P = 10x + 12y$$

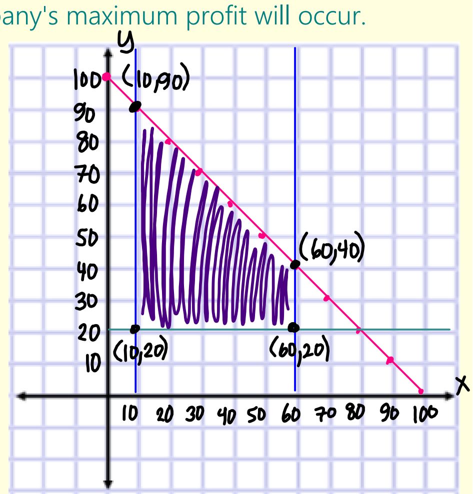
C. Graph the feasible region.

D. Find when the company's maximum profit will occur.

$x + y \le 100$
$-x \quad -x$
$y \le -x + 100$
$m = -1 \quad b = 100$
Solid / shade ↓

$10 \le x \le 60$
vert. lines/solid/
shade in between

$y \ge 20$
hor. line/solid/
shade above



$P = 10x + 12y$

(10,20)	(10,90)	(60,20)	(60,40)
$P = 10(10) + 12(20)$	$P = 10(10) + 12(90)$	$P = 10(60) + 12(20)$	$P = 10(60) + 12(40)$
$P = 100 + 240$	$P = 100 + 1080$	$P = 600 + 240$	$P = 600 + 480$
$P = 340$	$\boxed{P = 1180}$ max	$P = 840$	$P = 1080$

$\boxed{\text{10 afghans, 90 sweaters}}$

EXAMPLE 6 $x = \#$ of large bookcases $y = \#$ of small bookcases

A carpenter makes bookcases in two sizes, large and small. It takes 6 hours to make a large bookcase and 2 hours to make a small one. The profit on a large bookcase is \$50, and the profit on a small bookcase is \$20. The carpenter can spend only 24 hours per week making bookcases and must make at least 2 of each size per week.

A. Write a system of inequalities to represent the constraints.

$$\begin{cases} 6x + 2y \le 24 \\ x \ge 2 \\ y \ge 2 \end{cases}$$

B. Write an objective function for the company's total profit, P, from the sales of ~~afghans and sweaters.~~ large & small bookcases.

$$P = 50x + 20y$$

C. Graph the feasible region.

D. Find when the carpenter's maximum profit will occur.

$6x + 2y \le 24$

$2y \le -6x + 24$

$y \le -3x + 12$

$m = -3 \quad b = 12$

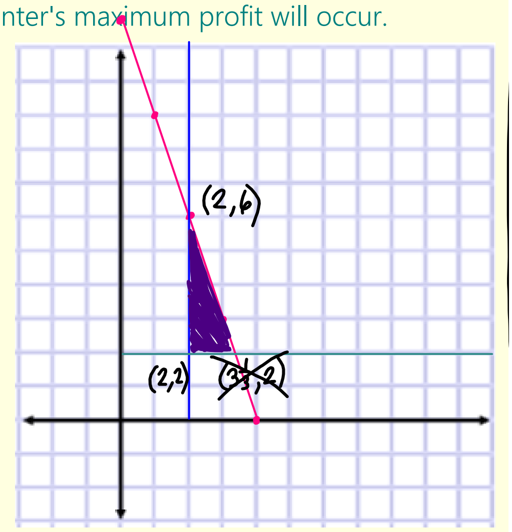
solid/shade ↓

$x \ge 2$

vert/solid/shade →

$y \ge 2$

hor/solid/shade ↑



$P = 50x + 20y$

(2,2)

$P = 50(2) + 20(2)$

$P = 140$

(2,6)

$P = 50(2) + 20(6)$

$\boxed{P = 220}$ max

2 large bookcases

6 small bookcases