

1.10 Lines

$$\text{Slope of a Line} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = m$$

Slope of a Vertical Line = *undefined*

Slope of a Horizontal Line = *zero*

Example 1

Find the slope of the line that passes through $(2, 1)$ and $(8, 5)$.

$$m = \frac{5 - 1}{8 - 2} = \frac{4}{6} = \frac{2}{3}$$

Point-Slope Form of a Linear Equation

$$y - y_1 = m(x - x_1)$$

Slope-Intercept Form of a Linear Equation

$$y = mx + b$$

Example 2

Find an equation of the line through $(1, -3)$ with slope $-\frac{1}{2}$ in slope-intercept form. Then sketch the line.

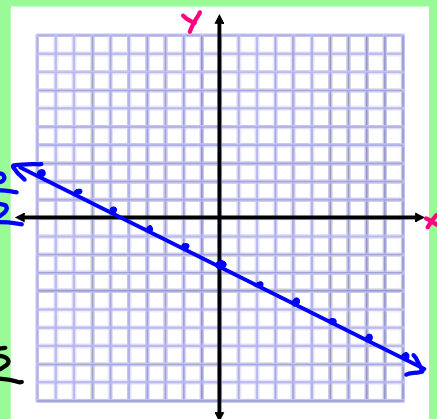
$$y - y_1 = m(x - x_1)$$

$$y - 3 = -\frac{1}{2}(x - 1)$$

$$y + 3 = -\frac{1}{2}x + \frac{1}{2}$$

$$\begin{array}{r} -3 \\ \hline y = -\frac{1}{2}x - \frac{5}{2} \end{array}$$

$$m = -\frac{1}{2} \quad y\text{-int} = -\frac{5}{2}$$



Example 3

Find an equation of the line through $(-1, 2)$ and $(3, -4)$ in slope-intercept form.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-4 - 2}{3 - (-1)}$$

$$m = \frac{-6}{4}$$

$$m = -\frac{3}{2}$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{3}{2}(x + 1)$$

$$y - 2 = -\frac{3}{2}x - \frac{3}{2}$$

$$+2 \qquad \qquad \qquad +2 \qquad +\frac{4}{2}$$

$$y = -\frac{3}{2}x + \frac{1}{2}$$

Example 4

Find the equation of the line with slope 3 and y-intercept -2 in slope-intercept form.

$$y = 3x - 2$$

Example 5

Find the slope and y-intercept of the line $3y - 2x = 1$.

$$m = \frac{2}{3}$$

$$y\text{-int} = \frac{1}{3}$$

$$\frac{3y}{3} = \frac{2x + 1}{3}$$

$$y = \frac{2}{3}x + \frac{1}{3}$$

Equation of the **Vertical Line** through (a,b) is

$$x = \#$$

$$x = a$$

Equation of the **Horizontal Line** through (a,b) is

$$y = \#$$

$$y = b$$

Standard Form of a Linear Equation

$$Ax + By + C = 0 \quad \text{or} \quad \boxed{Ax + By = C}$$

Example 6

Sketch the graph of the equation $2x - 3y - 12 = 0$ by finding the intercepts.

$$2x - 3y - 12 = 0$$

x-int
 $2x - 3(0) - 12 = 0$

$$2x - 12 = 0$$

$$2x = 12$$

$$x = 6$$

$$(6, 0)$$

y-int

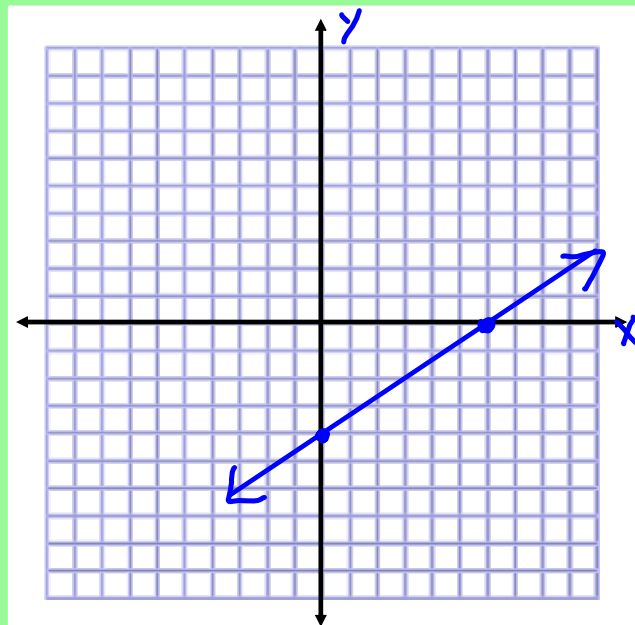
$$2(0) - 3y - 12 = 0$$

$$-3y - 12 = 0$$

$$-3y = 12$$

$$y = -4$$

$$(0, -4)$$



The slopes of **parallel lines** are always the same.

The slopes of **perpendicular lines** are always opposite reciprocals.

Example 7

Find the equation of the line through point $(5, 2)$ that is parallel to the line $4x + 6y + 5 = 0$.

$$\begin{aligned} 4x + 6y + 5 &= 0 \\ \frac{6y}{6} &= \frac{-4x - 5}{6} \\ y &= -\frac{2}{3}x - \frac{5}{6} \\ m &= -\frac{2}{3} \end{aligned}$$

$$\begin{aligned} y - y_1 &= m(x - x_1) && \text{same slope} \\ y - 2 &= -\frac{2}{3}(x - 5) \\ y - 2 &= -\frac{2}{3}x + \frac{10}{3} \\ +2 & && +2 + \frac{6}{3} \\ \boxed{y} &= \boxed{-\frac{2}{3}x + \frac{16}{3}} \end{aligned}$$

Example 8

Find the equation of the line through point $(-3, 7)$ that is perpendicular to the line $y = 4$.

vert. line \rightarrow hor. line

$$\boxed{x = -3}$$

When a line is used to model the relationship between two quantities, the slope of the line is the **rate of change**.

Example 9

A dam is built on a river to create a reservoir. The water level w in the reservoir is given by the equation $w = 4.5t + 28$, where t is the number of years since the dam was constructed and w is measured in feet.

a) Sketch a graph of this equation.

$$m = 4.5 = \frac{9}{2}$$

$$w\text{-int} = 28$$

b) What do the slope and w-intercept of this graph represent?

Water level is rising 9-ft every 2 years

rise
run

initial water level is 28ft

