

3.1 SYSTEM OF EQUATIONS

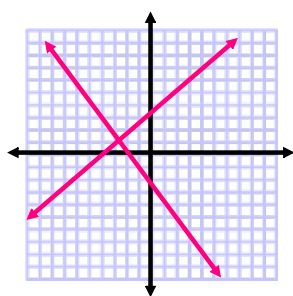
Two or more equations in
the same variables

A solution of a system of equations is any ordered pair that satisfies both equations.

There are 3 methods to solve systems of equations:

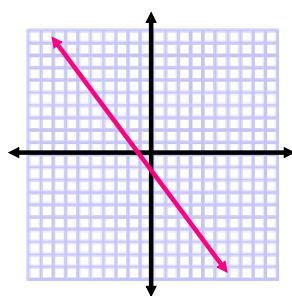
1. Graphing
2. Substitution
3. Elimination (Linear Combination)

Intersecting
Lines



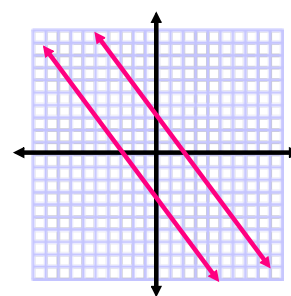
- Exactly one solution
- Consistent
- Independent

Coinciding
Lines



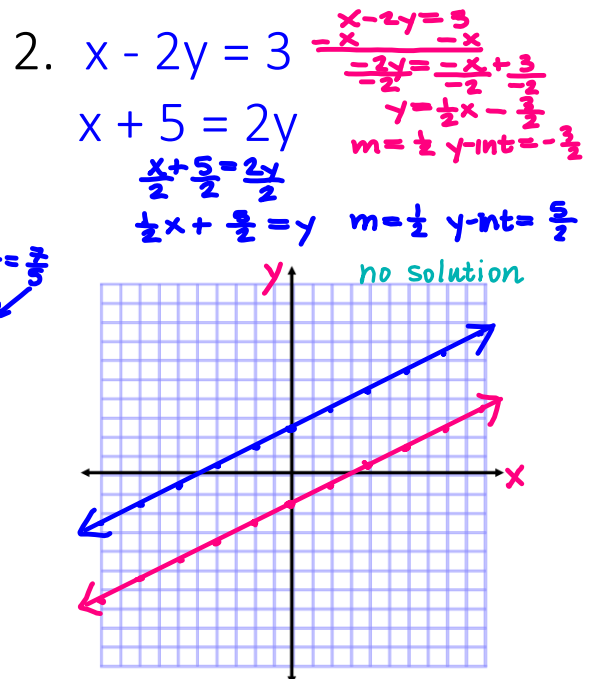
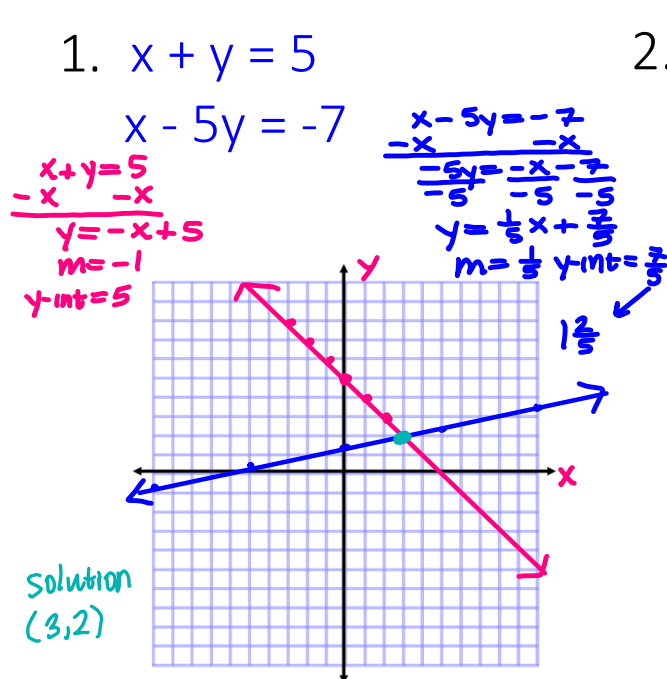
- Infinitely many solutions
- Consistent
- Dependent

Parallel
Lines

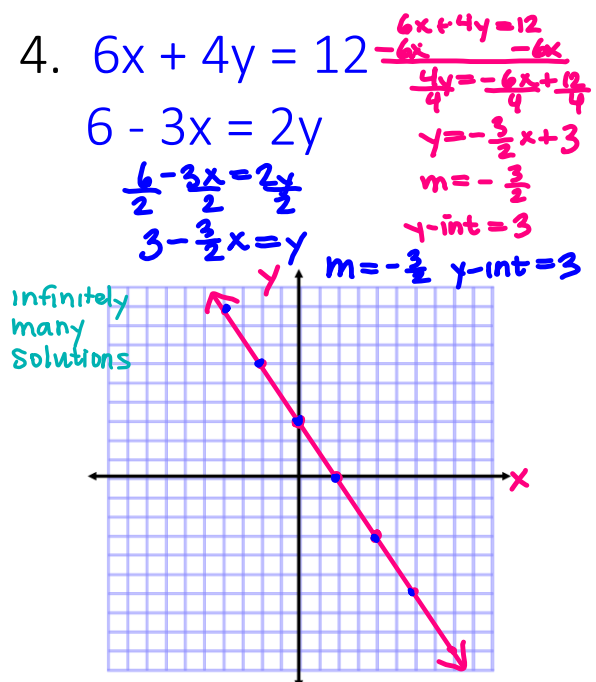
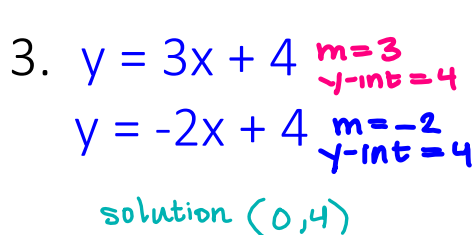


- No solution
- Inconsistent

Graph and classify each system.
Then find the solution from the graph.



Graph and classify each system.
Then find the solution from the graph.



SUBSTITUTION METHOD

STEPS

1. Solve one equation for one variable (choose the variable with no numerical coefficient).
2. Plug that equation into the other equation.
3. Solve for the remaining variable.
4. Plug that number answer into one of the equations to solve for the other variable.

$$\begin{aligned} 3x - y &= 13 \\ 2x + 2y &= -10 \end{aligned}$$

$$\begin{array}{r} 3x - y = 13 \\ -3x \quad -3x \\ \hline -y = -3x + 13 \\ -1 \quad -1 \quad -1 \\ \hline y = 3x - 13 \end{array}$$

$$\begin{array}{r} 2x + 2(3x - 13) = -10 \\ 2x + 6x - 26 = -10 \\ 8x - 26 = -10 \\ \quad +26 \quad +26 \\ \hline 8x = 16 \\ \frac{8x}{8} = \frac{16}{8} \\ x = 2 \end{array}$$

$$y = 3(2) - 13$$

$$y = 6 - 13$$

$$y = -7$$

$(2, -7)$

Use substitution to solve the systems. Check your solution.

$$\begin{aligned} 5. \quad 7x + y &= 6 \\ 5x - y &= -3 \end{aligned}$$

$$\begin{array}{r} 5x - y = -3 \\ -5x \quad -5x \\ \hline -y = -5x - 3 \\ -1 \quad -1 \quad -1 \\ \hline y = 5x + 3 \end{array}$$

$$\begin{array}{r} 7x + (5x + 3) = 6 \\ 12x + 3 = 6 \\ \quad -3 \quad -3 \\ \hline 12x = 3 \\ \frac{12x}{12} = \frac{3}{12} \\ x = \frac{1}{4} \end{array}$$

$$y = 5\left(\frac{1}{4}\right) + 3$$

$$y = \frac{5}{4} + \frac{12}{4}$$

$$y = \frac{17}{4}$$

$\left(\frac{1}{4}, \frac{17}{4}\right)$

$$\begin{aligned} 6. \quad 3x + y &= 8 \\ 18x + 2y &= 4 \end{aligned}$$

$$\begin{array}{r} 3x + y = 8 \\ -3x \quad -3x \\ \hline y = -3x + 8 \\ y = -3(-1) + 8 \\ y = 3 + 8 \\ y = 11 \end{array}$$

$$\begin{array}{r} 18x + 2(-3x + 8) = 4 \\ 18x - 6x + 16 = 4 \\ 12x + 16 = 4 \\ \quad -16 \quad -16 \\ \hline 12x = -12 \\ \frac{12x}{12} = \frac{-12}{12} \\ x = -1 \end{array}$$

$(-1, 11)$

7. $y = x + 3$ $y = 7 + 3$
 $y = 2x - 4$ $y = 10$

$$\begin{array}{r} x + 3 = 2x - 4 \\ -x \quad -x \\ \hline 3 = x - 4 \\ +4 \quad +4 \\ \hline 7 = x \end{array}$$

$(7, 10)$

8. $4x + 2y = 20$

$$\begin{array}{r} 4x + 2(x - 2) = 20 \\ 4x + 2x - 4 = 20 \\ 6x - 4 = 20 \\ +4 \quad +4 \\ \hline 6x = 24 \\ \frac{6x}{6} = \frac{24}{6} \\ x = 4 \end{array}$$

$y = x - 2$
 $y = 4 - 2$
 $y = 2$

$(4, 2)$

9. $x + y + z = 5$ \rightarrow $x + y + 2 = 5$ \rightarrow $x = 3 - y$ $x = 3 - 2$
 $2x - 3y + z = -2$ \rightarrow $2x - 3y + 2 = -2$ $x = 1$

$$\frac{4z}{4} = \frac{8}{4}$$

$z = 2$

$$\begin{array}{r} 2(3 - y) - 3y + 2 = -2 \\ 6 - 2y - 3y + 2 = -2 \\ -5y + 8 = -2 \\ -8 \quad -8 \\ \hline -5y = -10 \\ \frac{-5y}{-5} = \frac{-10}{-5} \\ y = 2 \end{array}$$

$(1, 2, 2)$

$$\begin{aligned}
 10. \quad & x - y - z = -4 \longrightarrow x - y - (-4) = -4 \longrightarrow \boxed{x - y + 4 = -4} \\
 & 5x + 2y - 3z = 7 \longrightarrow 5x + 2y - 3(-4) = 7 \longrightarrow 5x + 2y + 12 = 7 \\
 & \frac{6z}{6} = \frac{-24}{6} \\
 & z = -4 \\
 & x = y - 8 \\
 & x = 5 - 8 \\
 & x = -3 \\
 & (-3, 5, -4) \\
 & \begin{aligned}
 & \frac{x - y + 4 = -4}{x - y - 4 = -4} \quad \begin{array}{l} \text{+y} \\ \text{-4} \end{array} \\
 & \hline
 & x = -8 \\
 & 5(y - 8) + 2y + 12 = 7 \\
 & 5y - 40 + 2y + 12 = 7 \\
 & 7y - 28 = 7 \\
 & \quad \quad \quad \begin{array}{l} +28 \\ +28 \end{array} \\
 & \hline
 & 7y = 35 \\
 & \frac{7y}{7} = \frac{35}{7} \\
 & y = 5
 \end{aligned}
 \end{aligned}$$

$$\begin{aligned}
 11. \quad & x + y + z = 5 \\
 & 2x - 3y + z = -2 \\
 & 4z = -12
 \end{aligned}$$