2.6 PROVE STATEMENTS ABOUT SEGMENTS & ANGLES

To complete proofs, here are some helpful hints:

- Start with the given information and ask yourself if you can take that any further.
- Use the picture or diagram to help you.
- If you're stuck, look at what you're trying to prove and work backwards.

A theorem is a statement that can be proven.

REMEMBER...We can use the following in proofs:

- properties
- definitions
- postulates
- theorems

Theorem 2-1

Congruence of segments is

- reflexive
- symmetric
- transitive

REFLEXIVE PROPERTY $\overline{AB} \cong \overline{AB}$

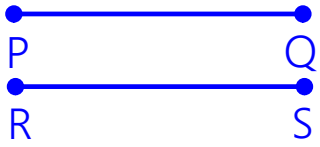
SYMMETRIC PROPERTY If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.

TRANSITIVE PROPERTY If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Let's prove the Symmetric Part of Theorem 2-1.

GIVEN: $\overline{PQ} \cong \overline{RS}$

PROVE: $\overline{RS} \cong \overline{PQ}$



STATEMENTS	REASONS
1. $\overline{PQ} \cong \overline{RS}$	1. Given
2. PQ = RS	2. Definition of congruent segments
3. RS = PQ	3. Symmetric Property
4. $\overline{RS} \cong \overline{PQ}$	4. Definition of congruent segments

Theorem 2-2

Congruence of angles is

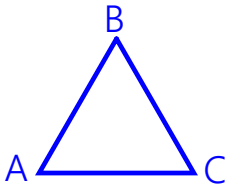
- reflexive
- symmetric
- transitive

Example 1

Use the diagram and given information to complete the missing steps and reasons in the proof.

Given: AB = 4, CB = 4, $\overline{CB} \cong \overline{CA}$

Prove: $\overline{AB} \cong \overline{CA}$

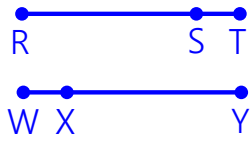


STATEMENTS	REASONS
1. AB=4, CB=4	1. Given
2. AB = CB	2. substitution prop.
3. $\overline{AB} \cong \overline{CB}$	3. def. of $\cong$
4. $\overline{CB} \cong \overline{CA}$	4. Given
5. $\overline{AB} \cong \overline{CA}$	5. Transitive Property

Example 2

Given: $\overline{RT} \cong \overline{WY}$, $ST = WX$

Prove: $\overline{RS} \cong \overline{XY}$

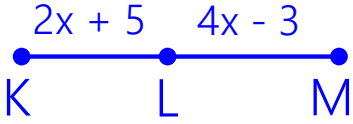


STATEMENTS	REASONS
1. $\overline{RT} \cong \overline{WY}$	1. Given
2. $RT = WY$	2. def. of $\cong$
3. $RT = RS + ST$ $WX + XY = WY$	3. Segment Addition Postulate
4. $RS + ST = WX + XY$	4. subst. prop
5. $ST = WX$	5. Given
6. $RS + ST = ST + XY$	6. subst. prop.
7. $RS = XY$	7. Subtraction Prop.
8. $\overline{RS} \cong \overline{XY}$	8. def of $\cong$

Example 3

Solve for the variable using the given information. Explain your steps.

Given: $KM = 38$

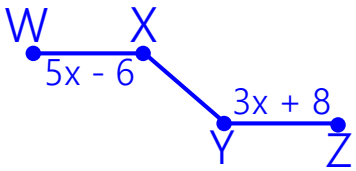


STATEMENTS	REASONS
① $KM = 38$	① given
② $KL + LM = KM$	② seg. add. post.
③ $(2x+5)+(4x-3) = 38$	③ subst. prop.
④ $6x + 2 = 38$	④ simplify
⑤ $6x = 36$	⑤ subtraction prop.
⑥ $x = 6$	⑥ division prop.

Example 4

Solve for the variable using the given information. Explain your steps.

Given: $\overline{WX} \cong \overline{XY}$, $\overline{XY} \cong \overline{YZ}$



STATEMENTS	REASONS
① $\overline{WX} \cong \overline{XY}$, $\overline{XY} \cong \overline{YZ}$	① given
② $\overline{WX} \cong \overline{YZ}$	② transitive prop.
③ $WX = YZ$	③ def. of $\cong$
④ $5x - 6 = 3x + 8$	④ subst. prop.
⑤ $2x - 6 = 8$	⑤ subtraction prop.
⑥ $2x = 14$	⑥ add. prop.
⑦ $x = 7$	⑦ div. prop.

Example 5

Given: Points P, Q, & S are collinear (in that order)

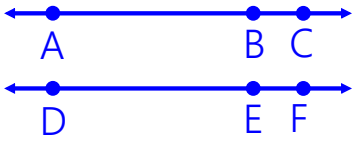
Prove: PQ = PS - QS

STATEMENTS	REASONS
① P, Q, & S are collinear	① given
② $PQ + QS = PS$	② seg. add. post.
③ $PQ = PS - QS$	③ subtraction prop.

Example 6

Given: $\overline{AC} \cong \overline{DF}$, $\overline{AB} \cong \overline{DE}$

Prove: $\overline{BC} \cong \overline{EF}$

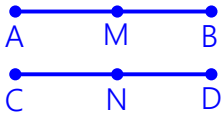


STATEMENTS	REASONS
① $\overline{AC} \cong \overline{DF}$, $\overline{AB} \cong \overline{DE}$	① given
② $AC = DF$, $AB = DE$	② def. of $\cong$
③ $AB + BC = AC$ $DE + EF = DF$	③ seg. add. post
④ $AB + BC = DE + EF$	④ subst. prop.
⑤ $AB + BC = AB + EF$	⑤ subst. prop.
⑥ $BC = EF$	⑥ subtraction prop.
⑦ $\overline{BC} \cong \overline{EF}$	⑦ def. of $\cong$

Example 7

Given: M is the midpoint of AB
N is the midpoint of CD
AB = CD

Prove: AM = CN

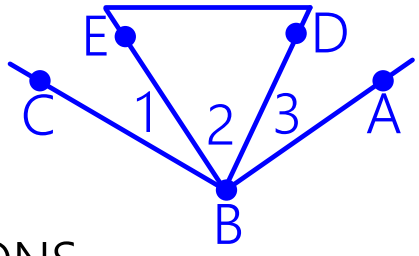


STATEMENTS	REASONS
① M is mdpt of $\overline{AB}$ N is mdpt of $\overline{CD}$ $AB = CD$	① given
② $AM = MB$, $CN = ND$	② def. of mdpt
③ $AM + MB = AB$ $CN + ND = CD$	③ seg. add. post.
④ $AM + MB = CN + ND$	④ subst. prop.
⑤ $AM + AM = CN + CN$	⑤ subst. prop
⑥ $2AM = 2CN$	⑥ simplify
⑦ $AM = CN$	⑦ div. prop.

Example 8

Given: $m\angle 1 = m\angle 3$

Prove: $m\angle EBA = m\angle DBC$



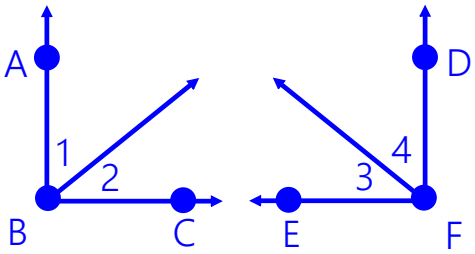
STATEMENTS	REASONS
① $m\angle 1 = m\angle 3$	① given
② $m\angle EBA = m\angle 2 + m\angle 3$ $m\angle DBC = m\angle 1 + m\angle 2$	② angle add. post.
③ $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	③ add. prop.
④ $m\angle DBC = m\angle EBA$	④ subst. prop.
⑤ $m\angle EBA = m\angle DBC$	⑤ symmetric prop.

Example 9

Given: $m\angle ABC = m\angle DFE$

$m\angle 1 = m\angle 4$

Prove: $m\angle 2 = m\angle 3$



STATEMENTS	REASONS
① $m\angle ABC = m\angle DFE$ $m\angle 1 = m\angle 4$	① given
② $m\angle ABC = m\angle 1 + m\angle 2$ $m\angle DFE = m\angle 3 + m\angle 4$	② angle add. post.
③ $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	③ subst. prop.
④ $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 1$	④ subst. prop.
⑤ $m\angle 2 = m\angle 3$	⑤ subtraction prop.